Survey of India

GEODETIC TRIANGULATION

BY

CAPTAIN G. BOMFORD, R.E.,

SUPERINTENDENT, SURVEY OF INDIA.

This pamphlet forms part I of the Handbook of Professional Instructions for the Geodetic Branch (Third Edition).

PUBLISHED UNDER THE DIRECTION OF

BRIGADIER R. H. THOMAS, D.S.O.,

SURVEYOR GENERAL OF INDIA.

PRINTED AT THE GEODETIC BRANCH OFFICE,
SURVEY OF INDIA, DEHRA DUN, 1931.

Price Two Rupees and Eight Annas or Four Shillings and Six Pence.

Survey of India

GEODETIC TRIANGULATION

BY

CAPTAIN G. BOMFORD, R.E.,

SUPERINTENDENT, SURVEY OF INDIA.

This pamphlet forms part I of the Handbook of Professional Instructions for the Geodetic Branch (Third Edition).

PUBLISHED UNDER THE DIRECTION OF

BRIGADIER R. H. THOMAS, D.S.O.,

SURVEYOR GENERAL OF INDIA.

PRINTED AT THE GEODETIC BRANCH OFFICE,
SURVEY OF INDIA, DEHRA DUN, 1931.

Price Two Rupees and Eight Annas or Four Shillings and Six Pence.

PREFACE

The last edition of this part of the Handbook of Professional Instructions was printed in 1902 as Part I of the Trigonometrical Handbook. The lapse of time, and especially the introduction of the Wild theodolites, has caused it to become out of date in many respects. Most of the changes in procedure now recorded have been introduced during the last 20 years by Dr. J. de Graaff Hunter, who has given me the benefit of his advice throughout its preparation. Captain G. H. Osmaston has also provided valuable criticisms as the result of his recent three years' experience of geodetic triangulation.

Where applicable, extracts have been freely made from the previous edition.

G. BOMFORD.

CONTENTS

Section I

Arrangement of Triangles and Controls

Section II

Reconnaissance and Building of Stations

Section III

Observations

CONTENTS

SECTION IV

Computations

Appendices

List of Figures

GEODETIC TRIANGULATION

SECTION I

ARRANGEMENT OF TRIANGLES AND CONTROLS

1. Definitions.—Triangulation is the process of measuring the angles of a chain or network of triangles formed by stations marked on the surface of the ground. If the length of one side (known as the base) is obtained by direct measurement, it is clearly possible to compute the lengths of all the other sides. Further, if the position of one point (known as the origin), and the direction of one side, are known with reference to some system of co-ordinates such as the meridians and parallels of some more or less arbitrarily defined spheroid (approximating to the sea-level surface), it is possible to calculate the co-ordinates of all the other points.

According to its quality, triangulation is known as primary, secondary, and tertiary or minor. In primary triangulation the average triangular error will vary from about $0''\cdot3$ in the best work up to about $1''\cdot0$. In secondary triangulation it may be as great as $3''$, but the best secondary series are sometimes better than the worst primary. In the Survey of India, primary and secondary triangulation is termed geodetic and is carried out by the Geodetic Branch, while minor triangulation is carried out by the topographical parties.

Except where otherwise stated, this handbook describes the methods used in primary triangulation. Secondary triangulation is carried out in a similar manner, except that a smaller instrument may be used, and the standards may be relaxed according to circumstances.

Minor triangulation is described in the Handbook of Topography, Chapter III.

2. Objects.—The objects of primary triangulation are two :—

(*a*) To constitute a framework on which less precise triangulation may be based, which in turn may form a framework for topographical maps.

(*b*) To assist (in combination with observations for height above sea-level, latitude, longitude, azimuth, gravity, etc.) in determining the size and shape of the Earth, the distribution of matter of varying density within the Earth, and the relation of such distribution to the Earth's surface features : and to test the permanence or to measure the mobility of those features.

3. System of chains.—For the attainment of both these objects the triangulation of a large country will best take the form of series of triangles forming a grid or framework as shown in fig. 1.

These series will normally be straight, and customarily run north and south or east and west, although this latter condition is less a practical necessity than a relic of the days when "Meridian Arcs" were the only basis for the investigation of the Earth's figure.

The sides of the quadrilaterals into which the country is so divided are (in India) generally between 100 and 300 miles. Where necessary for topographical purposes the larger quadrilaterals have been, or will be, divided by chains of secondary triangles, so that except in frontier areas all places where a good topographical map is required will lie within about 60 miles of geodetic triangulation.

4. Bases and Laplace stations.—All observations are liable to error, with the result that the computed positions of points at the far end of a series are not perfect. In particular the triangulation tends to accumulate errors of scale and azimuth, arising from the fact that the length and azimuth of each side of the triangulation are derived from the length and azimuth of the preceding side. Consequently, although a single base will suffice for the computation of any continuous system of triangulation, nevertheless it is proper to control and check the accumulation of error by the provision of other bases at appropriate intervals. Similarly it is possible, and equally necessary, to control the accumulation of error in azimuth by astronomical observations for azimuth (and longitude)*. Stations at which azimuth is so controlled are known as Laplace stations.

5. Distance between controls.—The intervals at which bases and Laplace stations should be introduced depend on many factors, namely :—The accuracy of the triangulation, the accuracy of base measurement (including necessary base extension), the accuracy of the azimuth control at Laplace stations, the latitude (affecting azimuth control only), the comparative cost of improving the quality of the triangulation or of providing extra controls, and on the money, instruments and personnel actually available. With well-conditioned triangles, measured with the highest accuracy the interval should be between 200 and 500 miles, depending on the lengths of the bases, and on the average length of the sides of the triangulation (see Dept. Paper No. 12, *Geodesy*, J. de Graaff Hunter).

* A simple astronomical observation for azimuth is ruined by the fact that the local vertical, on which the levelling of the instrument depends, does not coincide with the normal to the spheroid of reference on which the computations are based. Astronomical observations for longitude, combined with the longitude brought up by the triangulation, determine the required component of the angle between the local vertical and the spheroidal normal and so enable the azimuth observations to be corrected.


INDIA
TRIANGULATION SERIES
Corrected to Sept. 1928
REFERENCES
Series of Triangulation
Base Line
Scale 1 Inch=250 Miles or 1/15,840,000
Miles 100 0 100 200 300 400 Miles
ARABIAN SEA
BAY OF BENGAL
CEYLON
ANDAMAN ISLANDS
NICOBAR ISLANDS
Mouths of the Indus
Mouths of the Ganges
Mouths of the Irrawaddy
INDUS R.
BRAHMAPUTRA R.
GANGES R.
Sutlej R.
Jumna R.
Narbada R.
Godavari R.
Kistna R.
Chindwin R.
IRRAWADDY
SALWEEN R.
MEKONG R.
HERĀT
KĀBUL
PESHĀWAR
KANDAHĀR
NASRATĀBĀD
QUETTA
LAHORE
DELHI
AGRA
KARĀCHI
Karāchi Base
Chach Base
Dehra Dūn Base
Sironj Base
Sonākhoda Base
Calcutta Base
Bīder Base
Vizagapatam Base
Bangalore Base
Cape Comorin Base
Mergui Base
CALCUTTA
BOMBAY
MADRAS
NĀGPUR
RANGOON
MANDALAY
BANGKOK
LHĀSA
Rakaposhi 25550
Kunjut 25460
K² 28250
Gasherbrum 26470
Nanga Parbat 26620
Kamet 25447
Nanda Devi 25645
Gurla Mandhata 25355
Dhaulāgiri 26795
XXX 26658
Gosainthān 26291
Mount Everest 29002
Makalu 27790
Kinchinjunga 28146
Namcha Barwa 25445
Darjeeling
Jalpaiguri
Shillong
Dacca
Chittagong
Akyab
Prome
Moulmein
Amherst
Mergui
Port Blair
Colombo
Galle
Trincomalee
Negapatam
Trichinopoly
Madura
Tuticorin
Pāmban Pass
Cape Comorin
Minicoy
Cochin
Beypore
Calicut
Mangalore
Kārwār
Marmagao
Belgaum
Bāgalkot
Raichūr
Bellary
Bangalore
Secunderābād
Bolārum
Cocanāda
Vizagapatam
False Point
Sambalpur
Diamond Harbour
Kidderpore
Jubbulpore
Hinganghāt
Mhow
Ahmadābād
Bhaunagar
Port Albert Victor
Porbandar
Okha Point
Navānar
Hanstal
Deesa
Kalianpur
Gwalior
Cawnpore
Lucknow
Fyzābād
Benares
Allahābād
Meerut
Dehra Dūn
Ludhiāna
Amritsar
Multān
Dera Ismail Khān
Rāwalpindi
Murree
Srīnagar
Jacobābād
Sukkur
Hyderābād
Bikaner
Jodhpur
Poona
Prince's Dock
Apollo Bandar
Dibrugarh
Myitkyinā
Bassein
Elephant Pt.
Pilakat Creek
Diamond I.
64° 68° 72° 76° 80° 84° 88° 92° 96° 100°
8° 12° 16° 20° 24° 28° 32°

Fig. 2

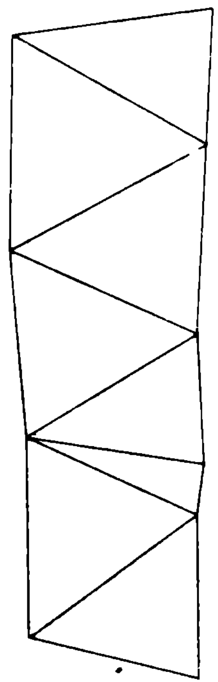


Fig. 2 (a)

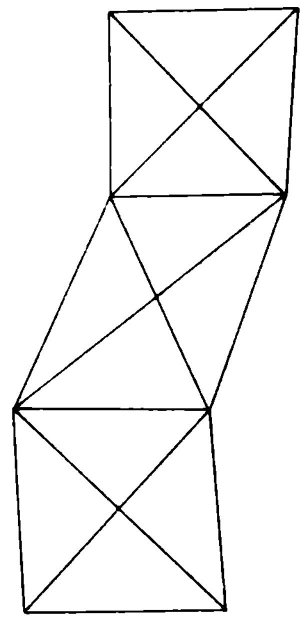


Fig. 2 (b)

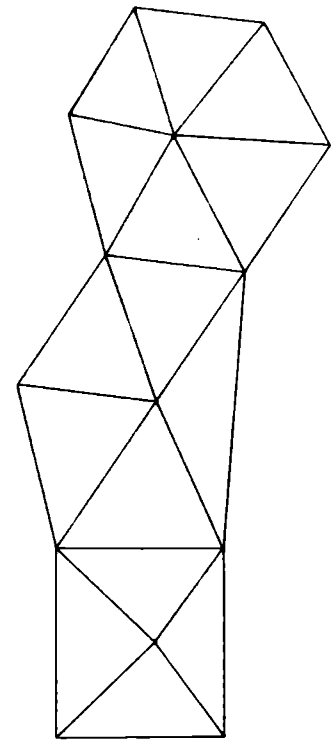


Fig. 2 (c)

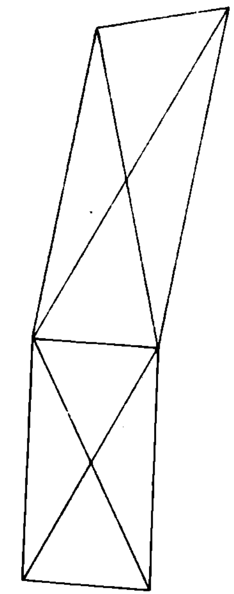


Fig. 2 (d)

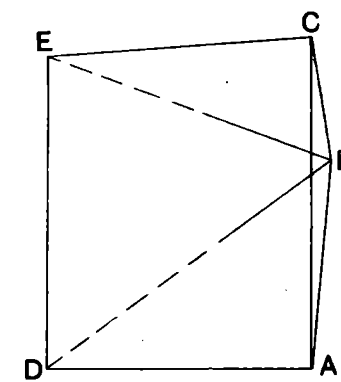


Fig. 2 (e)

Reg. No. 38 D.D.D. 1930 - 400

If local conditions, such as extensive flat plains, enforce a deterioration in the accuracy of the triangulation, the controls should be more frequent. The extreme case of close control is the primary traverse, in which the triangles are all replaced by directly measured traverse legs, and in which the azimuth should be controlled at about every 10 or 20 stations.

6. No direct control of position.—It might at first sight appear that astronomical observations of latitude and longitude would provide a check on accumulated error of position. But, as regards triangulation of high accuracy, this is not the case. Such observations are essential to the fulfilment of the objects of the triangulation [see para 2 (*b*)], but except for the indirect utility of the longitude observations at Laplace stations they do not control it.

7. Triangles, quadrilaterals, etc.—The arrangement of the triangles of a series must be suited to the nature of the country. A series may consist of simple triangles [fig. 2 (*a*)], of braced quadrilaterals [fig. 2 (*b*)], or of centered triangles, quadrilaterals, etc. [fig. 2 (*c*)]. Simple triangles generally cover the ground with the least number of stations; the siting of stations is simplified by the necessity of seeing only four other stations instead of the five required in a series of braced quadrilaterals; and the long diagonals of the latter are avoided. On the other hand, simple triangles do not give the check on the accuracy of the work, which is provided by more complex figures, nor the opportunity of avoiding a triangle which has turned out badly. In very flat country the avoidance of long diagonals may enforce a choice between triangles and centered figures such as hexagons, and considerations of cost may well result in the selection of simple triangles. But in hilly country, especially when the hills run in parallel ridges, it will often be almost as easy to lay out quadrilaterals as to lay out simple triangles. As a general rule, chains should be comprised of braced quadrilaterals, except where circumstances may make an occasional simple triangle difficult to avoid, or a more complex figure more convenient. An ill-conditioned quadrilateral should not be preferred to a pair of well-conditioned triangles, but in such a case as the second quadrilateral of fig. 2 (*b*) the weak diagonal should be observed if possible, in order to provide an independent check.

Ideally, simple triangles should be equilateral, quadrilaterals should be square, and all other figures should be regular. In particular, no angle may be small, which in the course of computation will fall opposite a known side, from whichever end of the series the computations may be begun. In simple triangles, such angles should seldom, if ever, be less than 45°. With quadrilaterals this will be impossible but it should always be possible to compute through a quadrilateral by at least one route involving no such angle of less than 35°. In pentagons and hexagons etc. this limit should be 40°.

A series such as that shown in fig. 2 (*d*) is very weak from the point of view of carrying forward the length of the sides, although it is good for azimuth, and will often be cheap and easy to observe. If it can be associated with a closer interval between bases, it is probably the ideal lay-out in hilly country with difficult communications. In any considerable extension of the Indian triangulation, such as a connection with Europe through Persia, it may be desirable to lay out figures with this object in view. But when, as at present (1929), primary triangulation in India takes the form of comparatively small extensions and additions to the existing framework, and when a number of bases have already been measured and there is difficulty in introducing new bases wherever they might otherwise have been thought advisable, then triangulation must conform to the standards laid down above.

8. Acute angle sometimes permissible.—It must be noted that a triangle containing a very acute angle which is not opposite a known side [e.g., the third triangle in fig. 2 (*a*)] is not weak from the point of view of carrying forward the length of the sides: nor does it introduce azimuth weakness unless the short side is so short that reasonable error in centering will cause appreciable angular error, a condition which is likely to be satisfied if the side is not less than 3 miles long. On the other hand such a triangle is wasteful: it involves the same expenditure of time and accumulation of error as an ordinary triangle, while it makes little progress with the series. A series should only contain triangles of this shape when necessary to surmount particular difficulties.

Even if the short side is only a few hundred yards long, azimuth can be carried forward through the long sides only, and the angles opposite these sides should only be used for determining their length ratios.

Another case of an acute-angled triangle is referred to in para 10.

9. Lengths of sides.—A primary series should consist of figures as large as the features of the country admit without grazing rays or extreme difficulty of observation, by which arrangement the number of stations and the probability of accumulation of error will be least. Considerations of economy likewise impose the same restriction, inasmuch as the additional points required for topographical purposes can be obtained at less cost by means of secondary and minor triangulation, in which accumulation of error is a less serious consideration, the limits of inaccuracy being sufficiently controlled by the primary triangulation. No general rule can be applied with respect to the magnitude of triangles, which must necessarily vary with the configuration of the country. In a mountainous country it would be difficult and injudicious to select small triangles, and in a flat country it is impossible to make large ones. In hilly country 20 to 40 miles is a convenient distance, and at such distances helios and lamps can generally be seen without difficulty. In a level country such as Bengal the most favourable length of side may be 10 miles or less, and 15 miles will rarely be exceeded.

10. Grazing rays.—The accuracy of triangulation depends entirely on the assumption that the path of a ray of light contains no curvature in the horizontal plane. It is well known that all light passing through the atmosphere is curved in the vertical plane, on account of the change in the density of the air with height. Lateral refraction (i.e., curvature in the horizontal plane) will only occur when the air density differs on either *side* of the ray. Such conditions are clearly unlikely to persist for any length of time at a distance from the ground, but a ray grazing close to the ground (particularly to sloping ground) will be very liable to such disturbance. It is not possible to lay down any permissible tolerance; it can only be stated that grazes must be avoided so far as is at all possible. A ray in which intervisibility is only secured at the hours of high refraction, should not be admitted, although some such rays have been included in Indian primary series (see G. T. Vol. II, Appendix III). Appendix I gives two examples of the errors caused by grazes. In hilly country the avoidance of grazing rays should be a matter of no great difficulty, provided stations are placed on the tops of the highest hills, and provided the sides are not made unreasonably long. If a grazing ray * [AC in fig. 2(*e*)] cannot be entirely avoided by a better lay-out, the remedy is to place an extra station near the point where the graze occurs. In cases of doubt, such a station should be built, visited by the observing party *, and observed to from stations A and C. The direct ray AC should also be observed. If the observations and triangular error (see para 50) show that the graze is harmless, the direct ray can be accepted and the extra station will have been unnecessary. But if the graze should prove to be serious, the observer will be spared the great delay of returning and re-observing.

In flat country, close grazes are unavoidable. They can only be minimised by shortening the sides and by raising the theodolite on towers or trestles. In any case accuracy will fall off, and closer base and azimuth control will be necessary. Alternatively, recourse may be had to primary traverse.

* The angle ABC must be accurately measured. In addition to this, the ratio of BC to BA must be obtained by an observation to D or E. If CBA is nearly 180°, this ratio is sufficiently well obtained by a rough intersection of D or E.

It is important that any ray which is thought to be liable to lateral refraction, should form part of some lay-out other than a series of simple triangles: it may then be possible definitely to attribute closing errors to the weak angle, and to reject it.

11. Stations to be on tops of hills.—Unless it is quite impossible, stations must always be placed on the highest point of a hill, and on the highest hill of a group. In country with very large features the second condition may be inadmissible by reason of the distortion of the triangles which would result, or of the disproportionate amount of labour involved in reaching the highest peaks, but as a general rule a primary station should be on the highest point within five miles. Besides increasing the risk of grazing rays, non-observance of this rule will result in considerable difficulties when other work is being based on, or connected with, the primary series.

12. Primary traverse.—Reference has been made in para 5 and para 10 to primary traverse as a substitute for triangulation in flat country. The only traverse of this nature which has been carried out in India has been a secondary traverse in the Punjab (1908-09), and in the absence of experience rules cannot be laid down. The following is an outline of what will probably be found to be the best procedure.

Distances will be measured by invar wires in catenary; this is a method of great accuracy and fair speed. Unless the traverse forms part of a fairly close network it will probably be desirable to measure the legs twice over, as a precaution against accidental error. Azimuth will be carried forward by sides as long as circumstances permit, angular measures being made with all the care usually bestowed on the angles of primary triangles. In the absence of any suitable natural elevations or buildings, it will probably be best to observe from small portable towers about 10 feet high surmounted by signals about 10 feet higher. Rays some miles in length should then be obtainable without difficulty, and azimuth carried forward with reasonable accuracy. Azimuth should be controlled by Laplace stations at intervals of about every 20 sides.

It is not, of course, necessary that the lines measured should coincide exactly with the straight lines between adjacent stations. They may deviate some degrees provided the deviation is adequately measured. And if the station is on a building or steep ground, the measurement need not be carried up to the station: it may be terminated at a convenient point laid off from the station at right angles to the ray. Reference may be made to the U. S. Coast and Geodetic Survey "Manual of First Order Traverse".

13. Intersected points.—Intersected points are points to which observations have been made from two or more stations, but which have not been visited, and whose fixing is immaterial to the continuity of the series. In minor triangulation they form the basis of the plane-table survey, but this is not work which can be adequately or economically performed by the geodetic triangulators; and without special orders no intersected points should be fixed with this object in view. There are three classes of intersected points which should be fixed by geodetic triangulators :—

Firstly.—Stations of important minor or exploratory triangulations or traverses which have been previously carried out. The fixing of such stations, or even of their intersected points (if well defined), enables valuable adjustments to be made. Whenever possible, connection should be made with adjacent pairs of stations, to determine the scale and azimuth error accumulated in the old work. It is important that the identity of old stations should be well established, and that the degree and extent of any doubt should be clearly stated.

For the frequency of such connections, the triangulator should be guided by the principle that minor triangulation should, if possible, be attached to better work every 50 or 100 miles.

Secondly.—It may sometimes be possible to fix intersected points which may be expected to be more permanent than the best constructed triangulation stations. The fixing of such points is a matter of great importance in trans-frontier or uncivilised localities, where the inhabitants may wilfully destroy the stations. Intersected points of this type should also be provided in intervisible pairs, suitable for the connection of future work. In addition to permanence, sharpness of definition is of course necessary; for a triangulator will always be able to make use of satellite stations to connect his work to an inaccessible point such as the top of a temple, while a base consisting of two near and unmarked rounded hills will be of no use to him. The numbers of these intersected points will depend on circumstances. In well-administered country where stations are cared for by the local authorities, their provision is hardly necessary. In such places as Baluchistān or the North-West Frontier they should be more numerous than the primary stations. Their frequency will also depend very largely on how many suitable points present themselves.

Thirdly.—The fixing of the peaks of distant mountain ranges in country which is politically or otherwise inaccessible, requires accuracy of observation which is only likely to be achieved by geodetic triangulation. Of course, only well-defined peaks should be observed.

Intersected points of the first and second of the above classes should be observed as a matter of course. Observations to points of the third class should not be undertaken without reference to the Director, but the officer in charge of the triangulation party should previously draw the Director's attention to the possibility of this kind of work.

The identification of intersected points is often a matter of some doubt, and for this reason they should be observed from at least three stations whenever possible, so that a check is provided. The heights should be observed in all cases.

14. Irregular expedients.—The geodetic triangulator is ordinarily debarred from making use of the following expedients :—

(*a*) • Satellite stations, except where unavoidable owing to the use of trestles and masts (see para 22), or to avoid grazing rays.

(*b*) Pivot stations, i.e. unvisited points whose fixing is essential to the continuity of the series.

(*c*) Triangles of whose angles only two have been observed.

(*d*) Resections of all kinds.

(*e*) Connection through pairs of points whose mutual distance is derived by computation from their latitudes and longitudes, instead of from the usual solution of a triangle.

15. Summary of subsequent sections.—The process of triangulation involves three distinct sets of operations :—

(*a*) The reconnaissance of the series and the building of stations.

(*b*) The observation of the angles.

(*c*) The computations.

These processes are described in the subsequent sections of this handbook.

SECTION II

RECONNAISSANCE AND BUILDING OF STATIONS

16. Separate detachment.—The work of reconnoitering and building the stations requires a less skilful officer and a smaller establishment than does the observation of the angles. It is consequently carried out by a separate detachment working ahead of the observer. The reconnaissance may be performed during the season preceding the observations, or, as is more usual, concurrently with observation but preceding it by a few stations. The advantages of reconnaissance during the previous year are that the observer runs little risk of being held up by delay in reconnaissance. On the other hand concurrent reconnaissance is generally more convenient from the administrative point of view; it avoids the risk of a change of programme postponing the observations until a later year when the jungle clearing will have to be repeated; and in case of difficulty the officer in charge of the reconnaissance is in closer touch with his officer in charge. Every triangulator would wish to have the reconnaissance completed during the preceding year, but in practice concurrent reconnaissance often has to be tolerated. The time by which the reconnaissance detachment must precede the observer depends on the nature of country, varying from a few weeks in bare hilly country to some months in flat forest country.

Although it seldom calls for much technical skill, the reconnaissance is often more arduous than the observation of the angles, and is in many ways more difficult, especially in forest-covered country. Bad reconnaissance will certainly delay the observations, and will often vitiate their accuracy.

17. Duties of reconnoitering detachment.—The duties of the officer in charge of the reconnaissance are:—

(*a*) To select stations and intersected points in conformity with paras 7-11 and paras 13-14 of this handbook, amplified or modified by any special instructions which may have been given to him. It will generally be possible for the officer in charge to select many of the stations from the existing maps, before field work commences. He will also give definite instructions regarding the types of figure to be preferred (triangles, quadrilaterals, etc.), the general length of side to be aimed at, and the permissible limit for small angles.

(*b*) To identify the mark-stones of all old stations with which connection has to be made (see para 20).

(*c*) To verify *beyond doubt* that all necessary stations are intervisible, and that the rays are not unduly nor unnecessarily grazing. A close grazing ray should be immediately reported to the observer with full details, illustrated by sketches, describing the nearness of the graze, and what rearrangement of the series will be necessary to avoid it.

He should observe the angles of the series with a small theodolite, and should report them to the observer before the latter makes the final observations.

(*d*) To clear long grass and jungle from around the stations. It is desirable, and generally possible, to clear the jungle all round, so as to give a clear view in all directions. In very heavy jungle it will suffice to clear the immediate neighbourhood of the station up to a radius of (say) 15 yards, and beyond that to clear sufficient only for the helios to be used at any time of day on all the rays which have to be observed.

(*e*) To build all stations in conformity with para 22 of this handbook, and to hand them over to the local authorities (para 26).

Copies of the descriptions of the stations and auxiliary marks will be sent to the observer for verification.

(*f*) To inform the observer of the best route to each station, and to give him the fullest information about communications, transport, camping grounds, supplies and water.

(*g*) By arrangement with the local authorities, and by his tact in dealing with them, to ensure that they will readily offer supplies, labour and assistance to the observing party.

(*h*) To keep an accurate plane-table chart of all the stations and intersected points which he selects.

(*i*) To perform all the above duties in such good time that unexpected delays and difficulties, which will often occur, will not result in delay to the observing party.

18. Composition of the detachment.—The reconnoitering detachment will generally consist of an officer of the Class II or Upper Subordinate Service, with an establishment of about 15 menials, including a couple of masons or brick-layers. Jungle clearing, path cutting and station building will be done by local labour under his direction. He should carry a small theodolite and two or three helios, and the squad should include men with some knowledge of helio work; in hazy weather or in flat country, the intervisibility of stations can only be proved by showing a helio. The inclusion of a few men with a knowledge of signalling is very desirable, although none exist in the department at present.

19. Method of reconnaissance.—The reconnaissance is carried out on a plane-table. If it is likely to be complicated or difficult, the plane-table should be mounted with a blue-on-white print of the best available $\frac{1}{4}''$ map. The reconnoitering officer will usually commence work by visiting the previously fixed stations on which the triangulation is to be based. From these stations he will cut in on the map, and identify, not only the points which have previously been selected as likely to be suitable, but also any other points which may possibly be required, either as stations or intersected points. He will make very full notes of what is visible and what is not visible, irrespective of whether the information appears likely to be of use or not. He will next visit the points which seem likely to make good forward stations and will follow the same procedure there. If observation is proceeding concurrently with reconnaissance, he will start building the pillar and platform, and clearing the jungle, as soon as a station is fairly definitely selected. The fixing of the upper mark-stone (para 24) must be done under his supervision, but the completion of the platform and of the jungle-clearing may be left to a reliable menial: or, if communications are bad it may be advisable to leave to the observer the fixing of the mark-stones, and the handing over of the stations.

If reconnaissance is being carried out sufficiently in advance of the observations, it will be better to select all the stations first, and to return along the series, constructing them on the return journey.

The reconnoitering officer must be particularly careful not to let the series degenerate into smaller and smaller figures. If he anticipates difficulties ahead, he will do well, if time permits, to go forward and discover the solution of them before committing himself too far with the stations in rear, which may afterwards have to be changed in order to meet the difficulty.

He should try to select at least twice as many intersected points as the observer is likely to use. He should visit (or send reliable menials to) all old minor stations which require connection: if they cannot be marked by opaque signals, he must inform the observer that helios will be necessary.

When starting work on a new series the observer and his detachment will often assist in the reconnaissance of the first few stations until it is well started, when he will return and start to measure the angles.

20. Good connection.—It is essential that the connection with the old work should be accurately established, i.e. the mark-stones or auxiliary marks of the old stations should be found undisturbed, or at least their original position should be identified within about a couple of inches. If sides are short even more accurately strict identification is necessary. If the old stations cannot be identified the work must be based on some other stations. Before taking the field the triangulator must obtain instructions from the Director as to what action he should take if difficulty is found in establishing a connection. It will generally be necessary to persist until one is found.

21. Well-built stations.—It is most important that the stations of geodetic triangulation should be marked by permanent structures, so that the positions of the majority of them can be found and identified (if possible to within one inch) after an interval of 100 years. With this object in view the stations are carefully built according to standard patterns, and are handed over to the local authorities, who report on their condition every year. Considerations of speed, economy or difficulty of access should not lessen the care which is given to the building of the stations. The less accessible the site, the greater is the risk of destruction; and, if anything, the more care must be given to the original building.

22. Types of stations.—The three types of stations which have been most largely used in the Survey of India are (*a*) Tower stations, (*b*) Trestle stations and (*c*) Hill stations.

Tower stations are hollow brick towers sometimes 50 feet or more in height. They have been used for crossing the plains of India, and are described in G. T. Vol. II and in previous editions of this handbook. In future work in similar country they would probably be replaced by portable trestles, or triangulation might be replaced by primary traverse.

Trestle stations may be made of local timber, or special portable trestles may be carried. It is desirable that the theodolite should be supported on a structure which is so far as possible independent of the structure on which the observer stands. Descriptions of trestles built in 1874-75 for the South-East Coast Series, will be found in the General Report of the G. T. Survey for that year. The only portable trestle hitherto used in the Survey of India is the Hunter Tower. This trestle weighs less than one ton, and can be erected to a height of 60 feet in eight hours. The support of the theodolite is not quite independent of the rest of the tower, but the connection is through a system of stays and gimbals designed to minimise movements of the theodolite. It is illustrated

Kulamangalam Station, Madura Series, 1916-17.
Height of instrument 67 feet. Height of signal 100 feet.

and described in Records Vol. VII and in fig. 3 of this handbook. It was used in the Madura and Bāgalkot (secondary) Series in 1916-17, and in the Rangoon (primary) Series in 1926-27 (see Records Vol. XI and Geodetic Report, Vol. III). The triangular errors of triangles involving this trestle have never averaged less than one second, and it seems unlikely that angles will ever be measured in India from high trestles with the best primary accuracy. Nevertheless, the triangulation can be accurately carried forward provided closer base and azimuth control is provided, and in flat country this will generally be easy.

Trestles and portable towers have been extensively used in the United States Coast and Geodetic Survey, where their use has resulted in rapid and inexpensive work of fair accuracy, although not with triangular errors averaging much below one second. But in America transport is generally far easier than in India, and skilled mechanical labour more easily obtained. (See U. S. Coast and Geodetic Survey *Reconnaissance and Signal Building*, and see *Bulletin Geodesique*, April 1928, Annexe 12, for reference to the Bilby portable tower).

When a portable trestle is moved to another station, it will generally be necessary to provide a mast to carry the signal to which observations will be made. A suitable structure is the Hunter portable mast (see fig. 3; also Records Vol. XI and Geodetic Report, Vol. III). The mast is made in 10-foot sections, and it can be raised to 100 or even 150 feet in a few hours. It can be used as an opaque signal by day, or a helio may be directed from the ground to a mirror on top of the mast (see Geodetic Report, Vol. III, page 88, reproduced as Appendix II to this handbook). At night a powerful petrol lamp can be hoisted to the top. Owing to difficulty in aligning the lamp, no reflector can be used, but the lamps were seen at a distance of 20 miles in the Rangoon Series.

The pickets holding the guys of these masts must be very firmly fixed in the ground, as any give in the anchorage will change the centering. Inaccuracy and possible inconstancy of centering probably make it impossible to attain the highest precision of angular measurement with these masts.

The site of a trestle station must be permanently marked by a structure similar to that marking a hill station, as described below. It is neither necessary nor possible to centre either mast or trestle over the permanent mark, but the centering error must be accurately measured (distance and azimuth) so that corrections can be applied to the observed angles.

Trestles of local material will be built by the reconnoitering detachment. Portable trestles will generally be erected by the observing detachment.

Hill stations.—The hill station is the type of station most commonly built, and which should seldom be departed from.

Where the surface is composed of rock, a dot surrounded by a concentric circle is cut on the solid rock in situ : otherwise, a large stone similarly marked is buried in the ground. Over the mark a short cylindrical pillar of strong masonry is built to a convenient height not less than 18 inches, and of 40 inches diameter: on the upper surface of this pillar another mark-stone is inserted, and fixed truly vertically over the lower one: an intermediate mark-stone should also be embedded in the pillar if it is 3 feet or more in height. A masonry wall 18 inches thick is built surrounding this pillar, and separated 3 inches from it. Round this wall is built a platform of earth and stones. The dimensions of this platform should be at least as large as the observatory tent, and it will generally be found convenient to make it somewhat larger; 12 feet × 12 feet will prove a suitable size. The annular space round the central pillar is for the purpose of isolating the instrument and preventing any shake caused by the observer's movements; it should be filled in with earth or loose sand. The distance between the two mark-stones should be recorded, but all measurements and observations should be referred to the upper mark-stone, and are so recorded in the angle books. In alluvial country where stone is scarce, it will be better to mark the circle and dot on bricks, as flat dressed stone is likely to be stolen by the inhabitants of such districts for domestic purposes. This kind of station is known as Type A, and should be built whenever a 12-inch theodolite is used.

When a Wild theodolite is used the station should be as above but the pillar should be of only 30 inches diameter. This type is known as Type D.

In secondary triangulation the 18-inch masonry wall may be dispensed with. Type C.

23. Auxiliary Marks.—At all geodetic triangulation stations three auxiliary marks should be provided, by which the station may be identified if the original mark is lost. Each should consist of a triangle with a central dot, deeply cut on solid rock if possible, or failing this on large stones embedded in the ground. They should be between 10 and 50 feet from the station. Their distances from the station and from each other should be recorded to the nearest inch, and the angle at the station subtended between each of them and some other G. T. station, should be measured to the nearest few minutes, the theodolite being directed on to them by means of its sight.

The horizontal distances from the auxiliary marks to the station are of most use if the station is destroyed. The most convenient way of obtaining them is by pointing the theodolite at each in turn, reading the vertical angles, and at the same time stretching a tape from the axis of the theodolite to the mark.

As a further precaution the circular pillars should be built carefully with the mark-stone as the centre; so that if a large portion of the circumference is found intact, the centre may be accepted as the old station within an inch or two.

24. Centering of upper mark-stones.—The method of adjusting mark-stones is as follows:—Let the external part of the platform be built up to the intended height of the upper mark, and place upon it four heavy stones in such a manner that threads stretched diagonally between them may intersect near the centre. Adjust these threads to correspond with a plumb-line suspended over the lower mark, and when the coincidence is complete, mark the four exterior stones by pencil lines, or lines scratched with a knife. Arrangements must now be made for protecting these stones, either by covering them over, or appointing a man to watch each, while the pillar in the centre is being built up nearly to the level of the next mark-stone, which is then adjusted to correspond with the cross-threads, and fixed in cement.

25. Construction of protecting cairn.—When the last helio squad leaves the station they should erect over the pillar a well-built cairn of large stones, six feet in diameter and six feet high. In country where no stone exists, it must suffice to pile up a rather larger mound of well-beaten earth or old bricks. *Khalāsis* should be instructed to make well-shaped, symmetrical cairns, centered over the station mark. The necessary materials should be assembled near the station before the observing party leaves it, to ensure that an adequate cairn is eventually erected.

26. Handing over to local officials.—The stations must be handed over to the headman or senior official of the village in whose lands it stands. The transfer forms 1, 2 and 3 Transfer are made out and signed by the Survey Officer and local officials. One copy will be kept by the latter, and the other two copies will be sent to the Director, Geodetic Branch, who will keep one copy and send the other to the District Officer in whose district the village lies. If the observer is in the field, a copy should also be sent to him.

SECTION III

OBSERVATIONS

27. Types of theodolite.—The theodolites now used for primary triangulation in the Survey of India are:—

(1) 12-inch, two-microscope.
(2) 12-inch, three-microscope.
(3) Wild Precision, 5½-inch.

The 12-inch theodolites have done work of the highest accuracy, and are well proved instruments. The Wild theodolite has great advantages over them in ease of observation and transport, but has not yet (1929) been fully tried. It is hoped that the 12-inch will never have to be used again, but this cannot be stated for certain, and rules for the use of both types of instrument are now given.

The two types of 12-inch theodolite differ only in the number of microscopes provided for reading the horizontal circle. In the three-microscope theodolite, a change of face results in the microscopes covering a different part of the circle; in the two-microscope theodolite it does not. With the same number of measures of each angle the former instrument will naturally give the greater accuracy, but the readings will of course take longer. On the whole there is little to choose between the two types.

An officer employed on primary triangulation may be presumed to be well acquainted with the construction of small theodolites as described in the Handbook of Topography, Chapter III, Appendix I. Except for their weight and accuracy the 12-inch theodolites differ little from the small micrometer theodolites there described. The principal differences are the provision of a moving wire eye-piece micrometer, by which repeated intersections may be made if the mark is unsteady, and the use of a much heavier braced stand, which cannot be folded up.

28. Adjustment of 12-inch theodolite.—The adjustments of a 12-inch theodolite are as follows:—They are described in detail in paras 29-38 and para 47.

(*a*) Centering, and verification of the plummet.
(*b*) Levelling, and adjustment of the levels.
(*c*) Adjustment of the transit axis.

(*d*) Adjustment of the limb microscopes for:—
(1) Distinct vision of the moving wire.
(2) Tangency to circle.
(3) No parallax between wires and image of circle.
(4) Run.
(5) Adjustment of comb zero.
(6) Zero reading on drum when zero of comb is bisected.

(*e*) Adjustment of the telescope for:—
(1) Distinct vision of the cross-wires.
(2) No parallax between cross-wires and image of a distant object.
(3) Collimation in azimuth.
(4) Verticality of the vertical wires.

(*f*) Collimation in altitude.

(*g*) Determination of the value of one division of the eye-piece micrometer.

(*h*) Determination of the value of one division of the bubbles.

(*i*) Examination for stiffness of bearings or looseness of joints, and adjustment of the vertical axis.

When making adjustments it should be noted that whenever a coarse adjustment has to be made, the corresponding fine adjustment should first be set to the centre of its run, so that it is left with the greatest possible scope for movement in either direction.

29. Centering.—Centering is done with the plummet. It should always be carried out with the greatest care, i.e., to within one tenth of an inch. With long rays such accuracy may appear to be unnecessary, but with short rays it is necessary, and it is not difficult to attain: moreover, it serves to impress the need of accuracy upon the menial establishment, who are responsible for centering the helios and lamps.

Occasionally it should be verified that the point of suspension of the plummet lies in prolongation of the vertical axis of the theodolite. If it does not do so, the point of the plummet will describe a small circle, when the theodolite is rotated. If the fault exists it should be remedied, or if this is not possible, the theodolite should be rotated at each centering, and the centre of the circle described should be made to coincide with the station mark. If the point of suspension does not rotate with the telescope, or if the centering is achieved by optical means (as in the small Wild theodolite), this test cannot be carried out. In such cases the adjustment must be verified from time to time by a small theodolite set up at a short distance (from two directions), whereby it can be seen whether the axis of the theodolite lies over the mark-stone, when it is apparently correctly centered.

If the point of suspension lies far from the plane of the three foot-screws, the centering must be verified after levelling. With optical centering it is of course essential that the instrument be fairly truly levelled before centering.

30. Levelling.—The operation of levelling is performed thus:—Let the level on the body of the instrument be placed parallel to a line joining two foot-screws, and, by means of these, bring the bubble to float in the centre of its tube. Then turn the instrument round 180° in azimuth, and if the bubble continues to float in the centre, that diameter of the limb which is parallel to the line joining the two foot-screws must be truly level: otherwise, half the difference of the readings at each end of the bubble is the error to be corrected by the foot-screws, and the other half difference of the readings is the error of the level itself. The latter error need not be corrected at all, if it amounts only to a few divisions, because it can always be allowed for in performing the adjustment. For instance, if the right hand end of the bubble reads a divisions, and the left hand end reads b divisions, in one position of the instrument, and after a semi-revolution in azimuth the same ends of the bubble read a' and b', then $\frac{1}{2}(a+a')$ and $\frac{1}{2}(b+b')$ are the readings which the bubble ought to have when the instrument is truly level. Now turn the instrument round 90° in azimuth and, by means of the third foot-screw, bring the bubble to read $\frac{1}{2}(a+a')$, $\frac{1}{2}(b+b')$. The diameter at right angles to the former one will now be approximately levelled, and if this operation is repeated two or three times, the adjustment will be perfected. The process should always conclude with the third foot-screw. If the error of the level bubble is very large, that is to say, if the reading of one end of the bubble differs considerably from the reading of the other, it can be rectified by the small capstan-headed screws attached to the level for that purpose. The proper time for effecting this correction is when the instrument is nearly level.

Small errors should not be corrected. Frequent tampering with the screws causes damage.

The criterion of the instrument's being level is that the reading of one end of the level remains constant during a complete rotation in azimuth. This should always be tested before beginning observations, and from time to time while they are in progress. The error of pointing, due to a dislevelment ζ at right angles to the line of sight, is $\zeta \tan h$, where h is the elevation of the point observed.

A dislevelment of one or two seconds is of no consequence when angles are being taken to terrestrial objects. When stars are being intersected, it will generally be necessary to record the readings of the bubbles, in accordance with such rules as are laid down for the observation in progress.

The adjustment of the bubble is generally controlled by two antagonising screws at one end of the bubble. It is important that the first turn should be given to that which has to be loosened, and that no great strength should be used; otherwise the threads will be stripped. On completion of the adjustment the antagonising screw must be tightened reasonably firmly, so that no slack is left between them.

31. Transit axis.—The telescope of every theodolite is supported on a horizontal axis the ends of which, called pivots, rest in angular supports called Y's. The line joining the centres of the pivots is the axis on which the telescope rotates, and in order that the latter may describe true verticals the axis must be truly horizontal. This adjustment is effected by raising or lowering one of the Y's by the appropriate screw placed beneath it for that purpose, the amount of adjustment being regulated by the indications of the striding level which is placed upon the pivots.

The error in the horizontal angle between two points of elevations h and h produced by a transit axis dislevelment of θ will be θ (tan h – tan h'). In the case of angles to terrestrial objects this is considerably less than θ. In any case it is completely cancelled by change of face, provided the body of the theodolite is correctly levelled. Consequently it is inadvisable to alter the adjustment of the transit axis for an error amounting to only 4 or 5 seconds, because frequent tampering with the screws will loosen them and destroy the permanence of the adjustment, the stability of which is more important than its temporary perfection.

It is necessary that the two legs of the striding level should be parallel to each other, for the vertical distance between one end of the bubble and the pivot on which that end rests, depends on the inclination of that leg to the vertical. Consequently, if the legs are not parallel, the indications of the striding level will vary with changes in its cross levelling. A small cross level is provided, and it should always be carefully centered. The parallelism of the two legs may be tested by seeing whether the readings of the striding level are sensitive to slight disturbance of the cross level If this is the case, the legs should be rendered parallel by means of the antagonising screws provided near the top of one leg, and the test repeated until the striding level is reasonably insensitive to errors of cross levelling. Imperfections in the internal grinding of the bubble tube will probably make it impossible to perfect this adjustment, and this consideration in itself provides strong reason for careful cross levelling.

The procedure is as follows:—Carefully level the theodolite. Then uncover the pivots and place the striding level gently upon them. Adjust it by the cross level, and read the end of the bubble nearest the vertical circle of the theodolite. Then reverse the striding level end for end, and again read the end of the bubble nearest the vertical circle. If the level is in good adjustment these two readings should be identical: if they differ by more than one or two divisions, the bubble should be brought to their mean by the capstan screws provided at one end of the bubble. When the level has been so adjusted that the reading at the end nearest the vertical circle is unchanged by reversal of the striding level, the bubble should be brought to the centre of its run by means of the capstan screw placed under one of the Y's of the theodolite.

This adjustment should be tested at every station, but it should seldom require correction.

32. Adjustment of the micrometers.—A scale reading of any kind demands the existence of a divided scale, an index mark whose reading is required, and a micrometer or vernier to subdivide the scale divisions. On the 12-inch theodolites the circle is divided into spaces of 5 minutes each. The rough reading to the nearest five minutes is made with a plain pointer in an auxiliary microscope with a large field. The reading to the nearest minute is easily made by counting on the comb visible in the field of A micrometer. The seconds (and decimals, by estimation) are read on the drum of each micrometer, one whole turn of which corresponds to one minute of arc. There are thus no less than seven* pointers (or index marks) all of which must be brought into approximate agreement as explained later.

(*a*) The pointer in the auxiliary microscope.

(*b*) The zeros of the combs in the three-micrometer-microscopes.

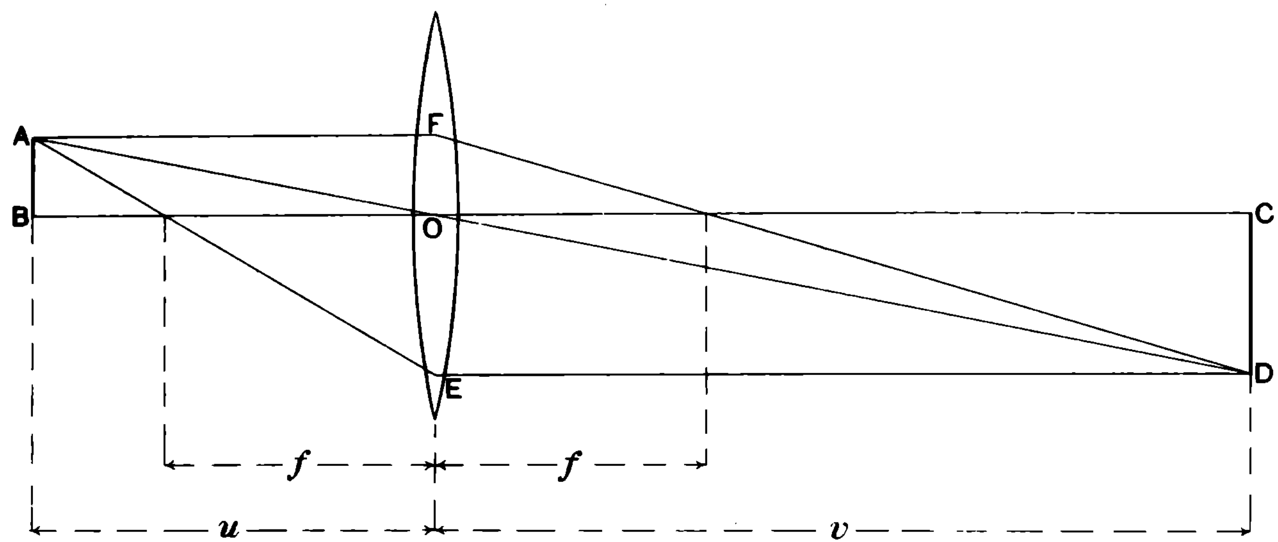
(*c*) The zeros on the drums of the three-micrometers.

The mean of the last three is the real "index mark" of the theodolite, whose position relative to the line of collimation of the telescope must remain unchanged during each single measure of an angle.

The first adjustment of a micrometer is for clear vision of the moving wires. This is a personal adjustment and is secured by moving the eye-piece in or out, while the circle is covered by a piece of white paper to provide a clear background for the wires. The second, third and fourth adjustments are inter-dependent and

* Five on a two-microscope theodolite.

Fig.4



Reg. No. 33 D.B.D. 1930 - 400.

must be carried out together. The second, namely making the movement of the micrometer tangential to the circle, is verified by bringing the moving wires over a long division of the circle; if the division bisects the wire throughout its length, the adjustment is correct. If not correct, it is remedied by bodily turning the microscope on its axis, either by slackening the collar which holds the tube of the microscope, or by slackening the three screws which fasten the microscope to the frame of the theodolite. If the run has not to be adjusted, the second method is preferable. The third adjustment, for absence of parallax, is corrected by moving the microscope bodily in and out. It is clear that the second adjustment must be verified immediately afterwards. The absence of parallax automatically secures clear vision of the circle when the eye-piece is adjusted for clear vision of the wires.

The fourth adjustment, for run, is the most troublesome of all the adjustments. It is clearly necessary that five turns of the micrometer drum should cause the moving wire to travel exactly from one graduation of the circle to the next. This is secured by regulating the magnification of the microscope. In fig. 4 the focal length of the lens (supposed thin) is f, the distance of the object from the lens is u, and that of the image is v. It is clear from the figure that the magnification is $\frac{v}{u}$, and that u and v are related by the formula:—

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

If by a bodily movement of the microscope the distance u is decreased, it is clear that v is increased, and that the magnification is altered. Thus two conditions have to be satisfied: firstly, the ratio $\frac{v}{u}$ should give the required magnification: and secondly, the distance v should be equal to the distance from the object-glass to the cross-wires, otherwise there will be parallax.

To secure the fulfilment of these two conditions, two movements are possible. The object-glass may be unclamped and screwed in or out of the body of the microscope thus changing both u and v, and the microscope may be moved bodily in and out, changing u only. It is by a combination of these two movements that the adjustment for run is made, the adjustments for parallax and tangency being continually verified.

The procedure is as follows. See that the first three adjustments of the micrometer are correct, and that the fifth adjustment (pages 22 & 23) is within the scope of the fine adjusting screws. Then with the moving wires intersect a graduation slightly to the left of the centre of the field, and read the micrometer drum. Then traverse the wires across two whole divisions (ten minutes), noting the direction of the rotation of the drum; intersect the second graduation and read again, noting by how much the rotation of the drum is in excess or defect of ten whole turns. If the rotation is in defect, the image is too small and it must be increased by protruding the object-glass from the microscope.

With a strange theodolite the observer will have no idea how much to move the object-glass. Then, if the run requires correction, he should turn the object-glass one full turn in the required direction, carefully readjust the microscope for clear vision of the limb by moving it bodily in or out, and re-measure the run. If the run was originally 5″ too great, and after one full turn it becomes 2″ too small, it is clear that one turn of the object-glass corresponds to 7″, and that the object-glass must next be turned back about one third of a turn. Two or three approximations will generally be necessary. It is important that the focus should be very perfect whenever the run is measured, for the run varies with the distance of the object-glass from the limb, and if the focus is bad a wrong value will be obtained for the value of one turn.

The graduations of the circle cannot be trusted to be everywhere equidistant, nor can all parts of the circle itself be trusted to be equidistant from the object-glass of the microscopes. Consequently, once the microscopes are in fair adjustment it is necessary to measure the runs in several different parts of the circle. Usually four places will suffice, two measures on each, but the triangulator must be guided by the differences he gets.

On no account should any attempt be made to improve the run at the expense of good focus; but when the run is very nearly correct, its final adjustment may be made by a bodily movement of the microscope, the object-glass being untouched, provided such movement does not result in appreciable parallax or poorness of definition.

All microscopes should be adjusted separately. The run of each should be correct within 1 second per 10 minutes. It is important that the final clamping of the object-glass should be sufficiently firm to prevent subsequent movement. If the object-glass is tightly clamped, its adjustment should remain unchanged throughout a season, and any correction necessary to the run will probably be correctly remedied by a bodily movement of the microscope.

The run should be tested, and if necessary adjusted, at every station. The readings should be recorded in the angle book, where any adjustments should also be recorded.

The fifth adjustment of the micrometers is to make the zeros of the combs agree with the pointer of the rough reading microscope within about half a minute. Micrometers B and C should be arranged to read about 30 seconds more than A, to avoid the chance of having to record different minutes when booking the angles. An adjustment of a few minutes can be made by a small screw on the left hand side of the micrometer box. Any larger adjustment can be made by slackening the three screws by which the microscope is fixed to the framework of the theodolite. At the same time the distance of the microscope from the axis of the theodolite should be so regulated that a convenient length of the graduations is visible in the centre of the field. Equal lengths should be visible in each microscope. If this coarse adjustment is necessary it should be carried out before the verification of run. The reading of the auxiliary microscope can itself be adjusted by means of two capstan screws near the eye-piece.

The last (sixth) adjustment of the micrometers is that the drum of the micrometer should read zero when the wires coincide with the zero of the comb. This may be corrected by holding the brass milled head with one hand, and turning the graduated head with the other: it is held by friction only.

The first four and the sixth of the above adjustments should be carried out for the vertical circle micrometers in a similar manner. The procedure for the adjustment of their combs is described with the adjustment of collimation in altitude in para 36.

33. Adjustment of the telescope.—Distinct vision of the cross-wires must be obtained by moving the eye-piece in or out. This is a personal adjustment, which may be tested and changed whenever convenient. When the eye-piece is being focussed, the telescope should be pointed at the sky or at a piece of white paper. Since the moving wire and the fixed wires cannot lie exactly in the same plane, it is not possible to get perfect focus on both together: the moving wire should be focussed in preference to the fixed.

It is next necessary to bring the image of the helio or other mark into the plane of the moving wire. This is done by a screw and rack which alters the distance from the object-glass to the diaphragm, or on some theodolites the rack moves an internal focussing lens. This focussing is independent of the personality of the observer, and it depends only on the distance of the object viewed. For all distances at which theodolite observations are normally made, the focus is sensibly the same, so that it is very seldom necessary to change the focus.

The adjustment is tested by intersecting a sharp distant * mark such as a steady helio, with the moving wire, and moving the head

* Distant at least one mile.

from side to side. If there is no parallax the mark remains intersected; if the object appears to move relative to the wires in the same direction as the head, there is said to be far parallax, and the distance between the object-glass and diaphragm must be decreased. If the object appears to move in the opposite direction there is said to be near parallax and the distance must be increased.

This adjustment must not lightly be changed. It must never be changed during the measure of an angle, nor between consecutive face right and face left measures, because it is apt to cause slight change in the line of collimation. It must never be changed merely in the hope of obtaining a clearer vision of the object; this is to be obtained with the eye-piece; if there is no parallax, one position of the eye-piece is best for both object and cross-wires. On the other hand, if parallax is found to be present, the focus must be changed: in the presence of parallax accurate observation is impossible. If the adjustment has to be made other than very occasionally, something is loose in the telescope and the instrument is defective.

A change in the distance between object-glass and diaphragm results in a change in the value of one division of the eye-piece micrometer. It has hitherto been the custom never to change the object-glass focus without re-determining the value of one division of this micrometer. This is not necessary. In the absence of parallax the distance between object-glass and diaphragm depends only on the figure and refractive index of the object-glass, and in the case of a theodolite telescope an error of focus which will cause appreciable error in a small micrometer reading, will cause very appreciable parallax. So that the test for parallax is sufficiently sensitive to ensure that the distance from object-glass to diaphragm is the same as it was when the value of one division of the micrometer was determined at the beginning of the season.

34. Collimation in azimuth.—The line of collimation of a telescope is the line joining the centre of the cross-wires and the centre of the object-glass. In a theodolite with a moving wire, the cross-wires in the above definition are replaced by the moving wire when it is in such a position that the mircrometer drum reads zero, and the wire itself is approximately in coincidence with the zero of the comb. When the telescope is rotated on its transit axis, the line of collimation should be at right angles to that axis, otherwise its prolongation will not describe great circles of the celestial sphere. Let TOT′ (fig. 5) be the transit axis, and let OB be the line of collimation making a small angle θ (AOB) with OA, the perpendicular to TOT′. This angle θ is called the collimation error. Then, on rotation, the line of collimation will describe a cone based on BZ′B′, and an object S will be referred to a point B on the horizon instead

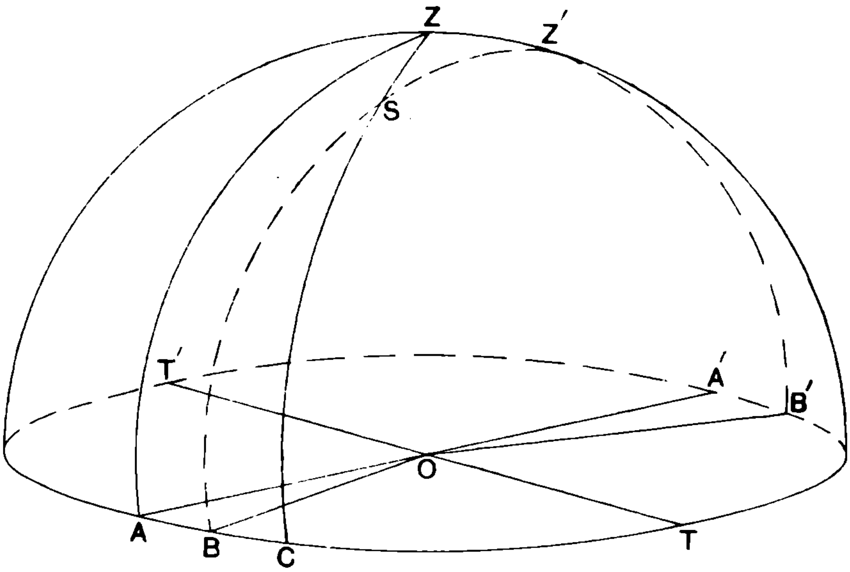


Fig. 5

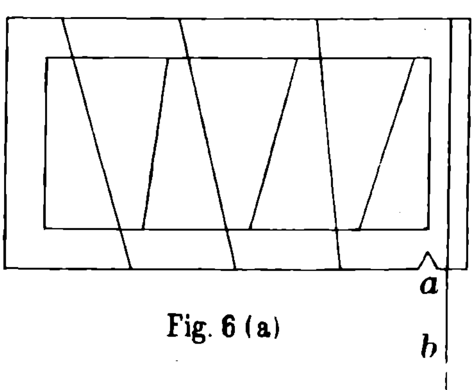


Fig. 6 (a)

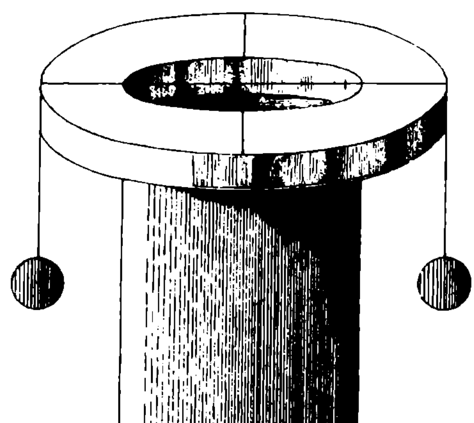


Fig. 6 (b)

Reg. No. 40 D.D.D-1930-400.

of to C. The error BC is then θ (sec h − 1) where h is the altitude of S. And the error in the horizontal angle measured between two points of elevations h and h' is θ (sec h − sec h'), a quantity which is very much smaller than θ in the case of observations between terrestrial objects, but which may be greater than θ if one of the objects is a star near the zenith. In practice, however, observations involving the horizontal angle between stars and other objects are (in Indian latitudes) never taken at an altitude of more than about 40°, in which case the error cannot exceed 0·3 θ. Further this source of error is completely cancelled by change of face, and the only objection to collimation error is that it causes systematic differences between readings on different faces, and so produces an appearance of inaccuracy. For observations to terrestrial objects there is no objection to as much as 30″ of collimation error in azimuth, although such a large amount is quite unnecessary. For astronomical observations it is desirable to reduce it to less than 3 seconds, but if any difficulty is experienced in so doing, it may quite well be left alone. As in the case of dislevelment of the transit axis, it is much more important that the error should be constant than that it should be small. Observations are of course always taken on both faces.

To eliminate the error of collimation in azimuth the telescope should be directed on a well-defined object near the horizon, and the circle read. If the moving wire is under consideration, it should first be set at zero. Face should be changed and the circle again read. Provided the transit axis has previously been levelled, half the difference of the two readings is the collimation error. If the object is on the horizon, this is the case whether the transit axis is level or not. The circle should be set to the mean reading, and the position of the vertical wire corrected. In the case of the moving wire this is readily done by the micrometer, the graduated drum of which should then be turned on its friction grip bearing, so that it reads zero while the mark remains bisected. The zero of the comb should also be brought into coincidence with the mark, by turning the small screw at the side of the micrometer box. In the case of the fixed wires, if it is desired to collimate one of them, the diaphragm must be moved sideways by means of the four capstan screws which hold it in place. Those at the top and bottom should first be slightly slackened off. When turning the horizontal screws it is important that the unscrewing movement should precede the screwing up, so that no damage is done to the threads.

The process should be repeated to verify that the adjustment is correct, and again repeated if necessary.

Collimation may also be corrected by Gauss's method using two auxiliary telescopes. The method is troublesome and open to the objection that it is almost impossible to collimate through the full aperture of the lens. It is described in Part VI (levelling) of this handbook.

This adjustment should be verified at every station, but should seldom require correction.

35. Verticality of the wires.—It is of course necessary that when the instrument is level, the vertical wires should be vertical. In practice, intersection will always be made close to the horizontal wire, but reliance should not be placed on this safeguard. To test the verticality of the wire the theodolite should be levelled, and some well-defined point intersected. The telescope should then be rotated in altitude, so that the mark appears to travel along the wire. By this means any crookedness of the wire, or lack of verticality is at once revealed. The former fault should be corrected by re-wiring (see para 39). In the case of the fixed wires lack of verticality should be corrected by slackening off the four screws which hold the diaphragm in place, and then turning the diaphragm. In the case of the moving wire, the adjustment is effected by turning the whole micrometer head by means of a tangent screw provided for the purpose. Especially in the case of the fixed wires this adjustment must be carried out concurrently with that for collimation in azimuth, the final correction being for verticality. The adjustment must be verified at every station.

The right angle between the grooves in the diaphragm cut for the horizontal and vertical fixed wires is fixed by the makers; they cannot be adjusted separately. The truth of the vertical wire is more important than that of the horizontal, and the adjustment should always be perfected for the former, except when all horizontal angles are to be measured with the moving wire only. Any appreciable error in the right angle is probably due to bad wiring, and can be corrected.

36. Collimation in altitude.—Although generally known as the adjustment for collimation in altitude, this adjustment is of quite a different nature from that for collimation in azimuth. Its only object is to ensure that vertical angles have the same values on face right and face left: for, provided there is no looseness between the vertical micrometers and the upper bubble, or between the vertical circle and the line of collimation, the mean of two faces is always correct. The first necessity is to adjust the upper bubble. The theodolite is levelled, as judged by the bubble remaining unmoved during a full rotation in azimuth. Then, if the bubble is not reasonably central, it is brought to the centre of its run by means of the antagonising screws at one end. It is not necessary that the bubble should be truly central, because bubble readings are always made and corrections applied. Next it is necessary that the zeros of the two vertical micrometers should be so adjusted that when the theodolite is truly levelled, and the theodolite is directed on an object, the mean of the two vertical micrometer readings, should be approximately the true elevation. To test this adjustment the elevation of a clearly defined mark is read on both faces: the mean corrected for bubble (see para 53)is the true elevation. The micrometers must then be so set that their mean reading equals this elevation: that which is read first should be set to read 15 seconds low, and the second correspondingly high. If the correction is only a few minutes, the zero of the comb can be moved to an approximately correct reading by means of the small screws in the end of the micrometer box, and the zero of the micrometer drum can be corrected by slipping it on its friction bearing until it gives the required reading while the correct graduation of the circle remains bisected. If the correction is larger it must be adjusted by means of the six antagonising screws at the base of the bracket which holds each microscope. These six screws, by tilting the bracket sideways, also provide the means of securing that a convenient length of the circle graduations is visible in the microscope. This coarse adjustment, if required, should be made before the other adjustments of the vertical microscopes (see para 32). Fine adjustments of the comb and micrometer drum should be made last.

Sometimes the vertical circle can be rotated relative to the telescope. On such theodolites it may be necessary to make a very large adjustment for zero, which is achieved by slackening the screw which clamps the circle.

An auxiliary wide field microscope is provided for reading the degrees. It also must be adjusted to give approximately the correct reading on both faces. It can be adjusted by means of two capstan screws near its eye-piece, or more coarsely by slackening off the three screws which hold it to the framework of the theodolite. In such a theodolite it will be noticed that when the telescope is horizontal, the line joining the zero graduations will not be horizontal, but will be inclined at such an angle as will bring it under the auxiliary microscope.

The collimation in altitude is automatically tested whenever vertical angles are observed. The collimation error should be recorded in the angle book at the beginning of the observations at each station. It is not necessary that the adjustment should be corrected unless the difference is so large as to lead to inconvenience in taking means. A measure of a vertical angle on one face is never deserving of consideration, while the mean of measures on two faces is independent of the adjustment.

The collimation in altitude should never be adjusted by moving the diaphragm and cross-wires of the telescope, although it is possible to correct it in this way.

37. Eye-piece micrometer.—The value of one division of the eye-piece micrometer in terms of seconds of horizontal arc must be determined at the commencement of the season at Dehra Dūn, and again at the close of the season. The determination of the value of one division of the micrometer is effected by moving the telescope through a small angle in azimuth and measuring the horizontal angle through which it is moved, both on the limb and with the micrometer.

An angle of about 30 seconds is the largest which the eye-piece micrometer will be called on to measure in practice, and it is consequently amply sufficient to determine the value of its divisions correct to about one part in a thousand. In view of the cancellation of error which occurs when the results of different measures are meaned, an error of even one part in a hundred is of little consequence.

The theodolite should first be adjusted for collimation in azimuth, so that the part of the screw measured will be that which will come into use in the field, and the circle micrometers should be carefully adjusted for run. The moving wire should then be moved five full turns to the left of the zero of the comb, and a fine mark intersected by means of the upper plate tangent screw. The eye-piece micrometer should then be read and recorded, as also should be the two (or three) circle micrometers. The moving wire should then be moved one full turn to the right, the mark re-intersected with the tangent screw, the circle read, and the process repeated, until it has reached five turns to the right of the zero of the comb. In the case of a two-microscope theodolite, zero should then be changed 45°; in a three-microscope theodolite it should be left unchanged. Face is changed and the observation repeated in the reverse direction. The zero of a two-microscope theodolite should then be changed a further 45°, and a three-microscope 30°, and the process repeated once more on each face, making a total of four measures altogether. The resulting angles corresponding to each position of the moving wire should then be tabulated, meaned, and examined for any irregularity. If there appears to be considerable systematic irregularity more readings should be made close to the zero, and a value determined from them only. In the absence of any irregularity the value should be deduced from the 1st and 10th, 2nd and 9th, and 3rd and 8th readings, and the simple mean of these three accepted.

The value of one division so obtained must be multiplied by the cosine of the altitude of the point observed, to reduce the angle to its horizontal value. Similarly, when angles are being read, the mean micrometer reading ought to be multiplied by the secant of the altitude, although the resulting correction is generally negligible (see para 55).

38. Bubble tester.—The value of one division of the scale of all the levels employed may be determined either at the Mathematical Instrument Office, Calcutta, or at the Geodetic Branch Office, Dehra Dūn, by means of the bubble tester, before the party takes the field and again after its return. Alternatively, and equally satisfactorily, it may be ascertained by fixing the level to the frame of the vertical circle, or making it ride parallel to the telescope, and then taking the readings of the microscopes in two positions of the bubbles. Whence, comparing the number of divisions of the level scale run over by the bubble with the corresponding angular motion of the vertical circle, as measured by the microscopes, the value of one division of the level scale will be obtained by simple proportion, as shown in the following example:—

Temperature.	Vertical Microscope			Angular Difference	Level					Computed value of one division of the level scale
	D	E	Mean		Readings		Differences			
					E	O	E	O	Mean	
Fahr. 70·7	7° 25′ 47″·0	41″·0	7° 25′ 44″·0	″ ...	d 85·6	d 48·4	d ...	d ...	d ...	″ ...
	7 26 20·5	14·1	7 26 17·3	33·3	50·9	83·0	34·7	34·6	34·7	·96
	7 26 12·1	6·0	7 26 9·1	8·2	59·9	73·9	9·0	9·1	9·1	·90
	7 25 62·1	56·2	7 25 59·2	9·9	70·1	63·8	10·2	10·1	10·2	·97
	7 25 52·2	45·0	7 25 48·6	10·6	81·0	53·1	10·9	10·7	10·8	·98
	7 25 44·2	37·3	7 25 40·8	7·8	89·9	44·3	8·9	8·8	8·9	·88
	7 26 11·1	4·9	7 26 8·0	27·2	61·2	73·0	28·7	28·7	28·7	·95
										5·64
									Mean value of one division of the level scale	0·94

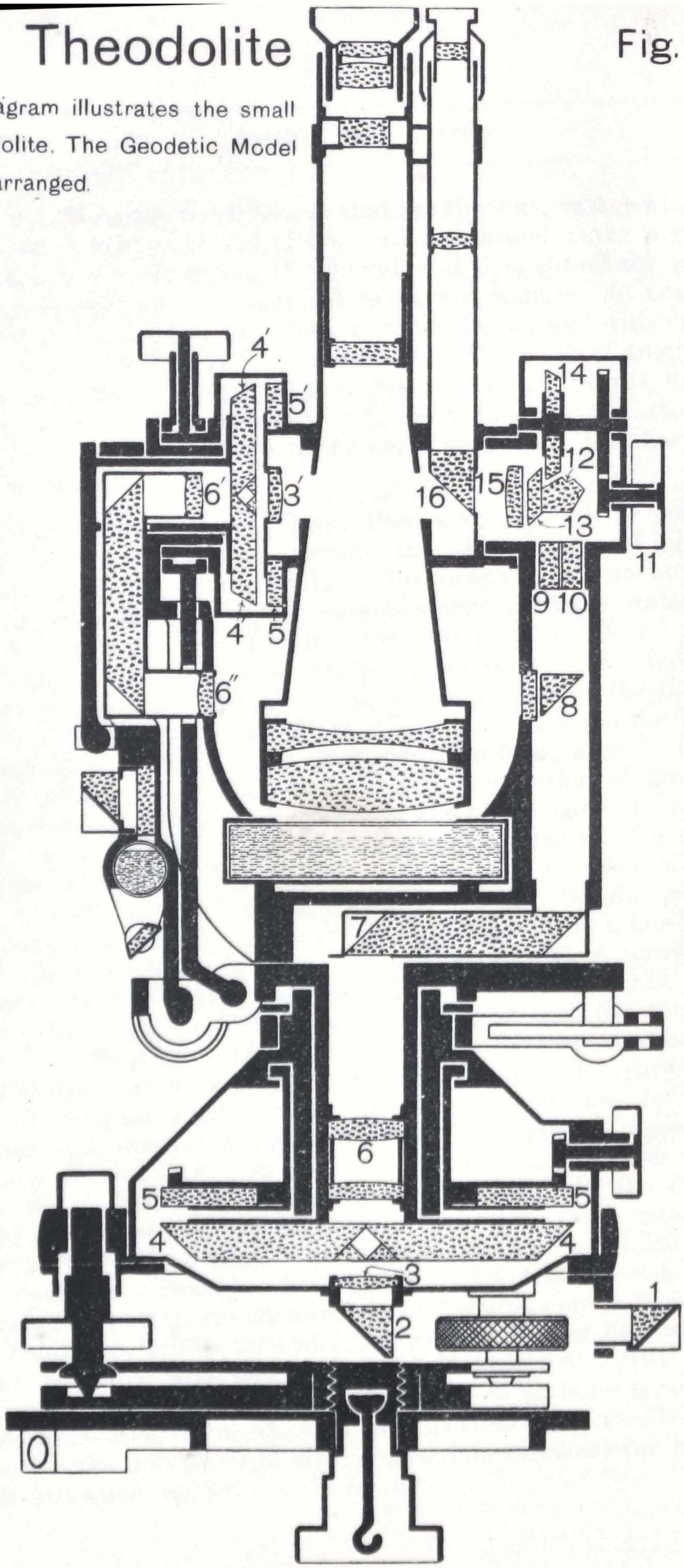
39. Re-wiring diaphragm.—As wires are liable to be broken, and to become slack by damp or uneven by accumulated dust, it is necessary that the triangulator should be able to replace them when required. The best substances for large instruments are spider lines, and for small ones, the fibres of raw silk. To procure spider lines, prepare some card frames, as shown in fig. 6 (a). Next find an active spider, and take it up on the edge of the card frame. Then gently shake the frame to detach the spider, which will hang from it. Wind up the fibre, so that the turns are rather wide apart, as shown. When the end of the card is reached, make a notch *a*, in which insert the end of the thread, and then cut off the rest with a pair of scissors. As several cards are wanted, do not let the spider fall to the ground, where it would be covered with dust, but take hold of the line at *b*, and place it on the notch of another card, and wind it up as before. The cards should afterwards be placed between leaves of clean paper, to preserve them from dust. Several such cards should be taken to the field, as in some localities suitable spiders cannot be found.

Having obtained the fibres, they may be fixed as follows:—Take out the diaphragm, remove the old wires, and clean the varnish from the engraved cuts. This must not be done by scraping with a knife, but by using methylated spirits and warm water, wiping off the old varnish until the cuts appear quite clean. This is important, as also is the correct thickness of the spider line: otherwise the lines may project above the surface of the diaphragm, and foul the moving wire. Now take a card, examine the fibre with a magnifying glass and select a clean uniform piece. Take two small balls of wax, attach them to the extremities of the selected fibre, and cut off the remainder. If one of the balls of wax is held in the hand, and the other is allowed to hang freely, the fibre will become straight and untwined. Now take a camel hair brush, dipped in clean water, and rub the fibre gently, for the purpose of cleaning and damping it. Put the diaphragm upon a block of wood and place the fibre upon it, examining it with a watch-maker's eyeglass to ensure that the wire falls into the proper cuts. The two wax balls hanging on either side serve to stretch the thread, and keep it in its place. The cross-wire is prepared in a similar manner, and as many more as may be required. Then taking care that they are all truly adjusted in their respective lines or cuts, let a drop of varnish fall upon each cut, and put a glass over the apparatus, to protect it from dust. In twenty-four hours the varnish will have set, the ends of the fibres may then be cut off, and the diaphragm carefully replaced in the telescope. This last is the most difficult part of the undertaking, because it requires very delicate handling to replace it without breaking a wire. The best varnish to use is copal, but sealing wax dissolved in methylated spirits, friar's balsam, or laudanum, will do instead. Previous damping of the fibres ensures their becoming tight and well stretched when dry. The best time to apply fresh wires is during the rains, when there is no dust, and the atmosphere is damp. It is important that the triangulator should take into the field all the materials necessary for performing this operation, viz.:—

Wild Theodolite

Fig. 7

This diagram illustrates the small Wild Theodolite. The Geodetic Model is similarly arranged.



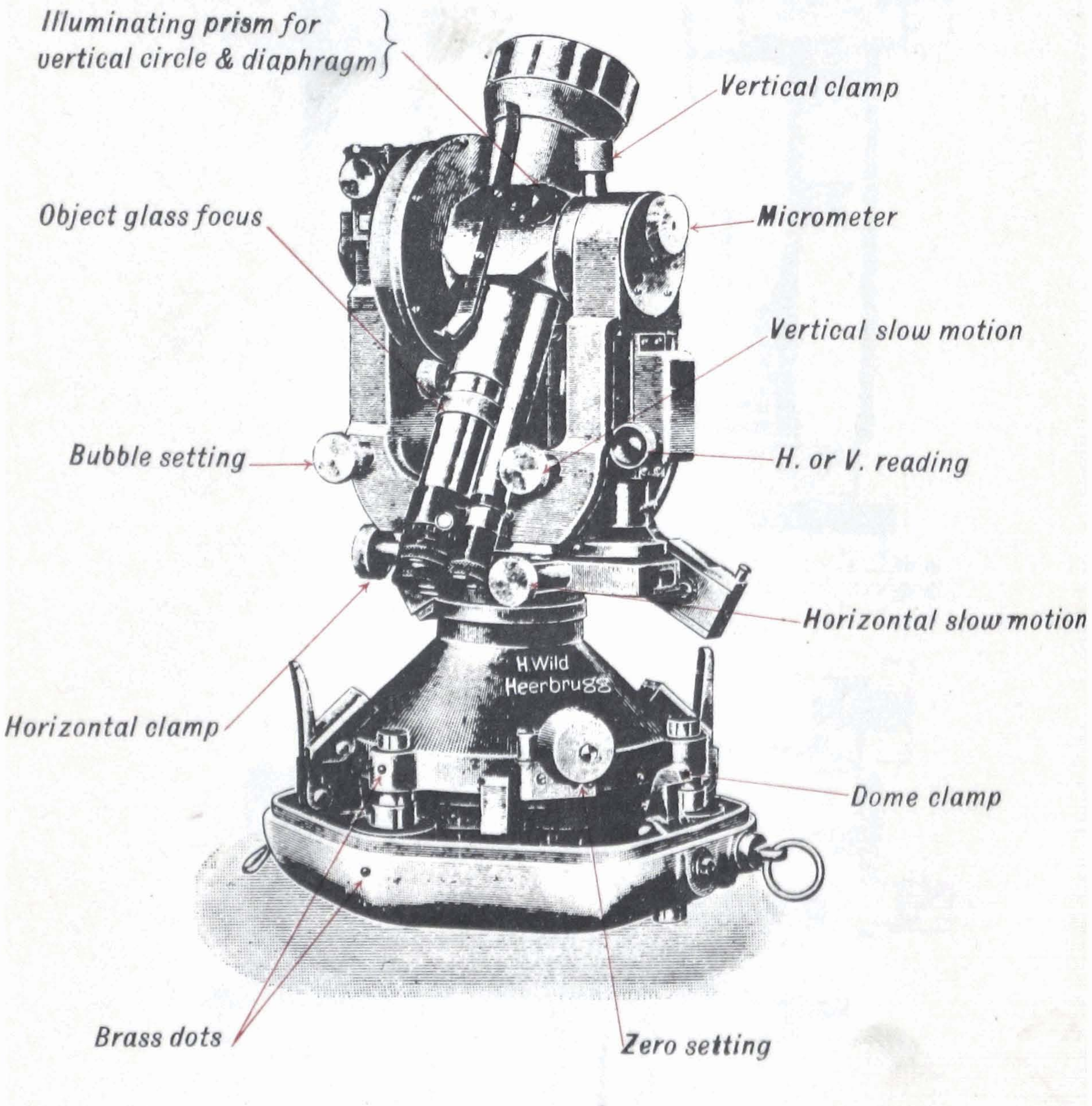
Reproduced from Messrs. Wild's diagram

Reg. No. 41 D.D.D 1930-400.

Fig. 8

Wild Theodolite

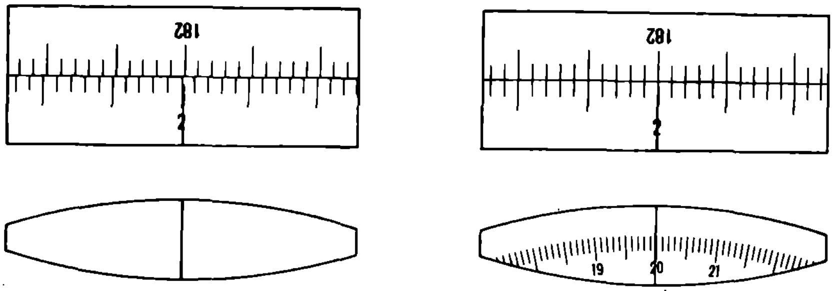
Geodetic Model



Reproduced from Messrs. Wild's Pamphlet
"Präzisions-Theodolite Wild"

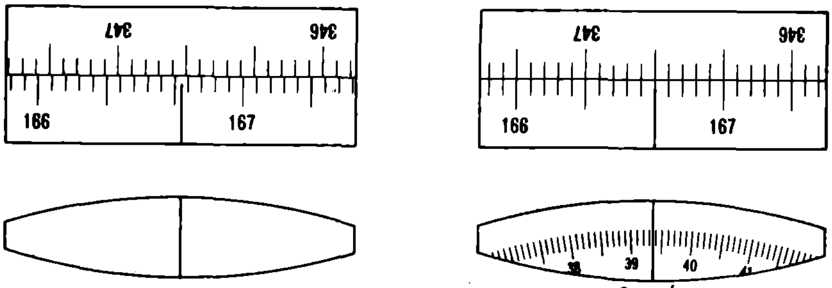
Fig. 9

Before Coincidence | After Coincidence



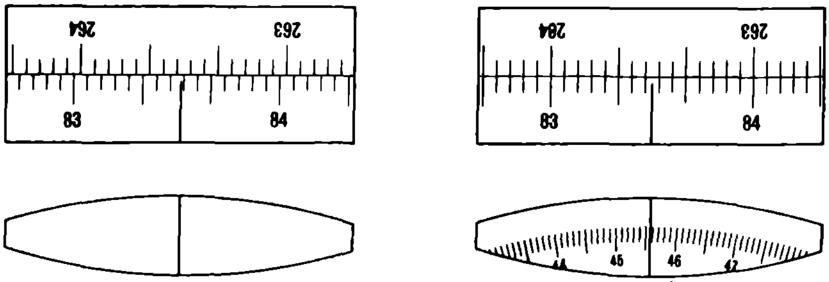
Horizontal Circle

	Circle reading	2° 0′
1	Drum reading	20″·0
2	Drum reading	19·9
	Total reading	2° 0′ 39″·9



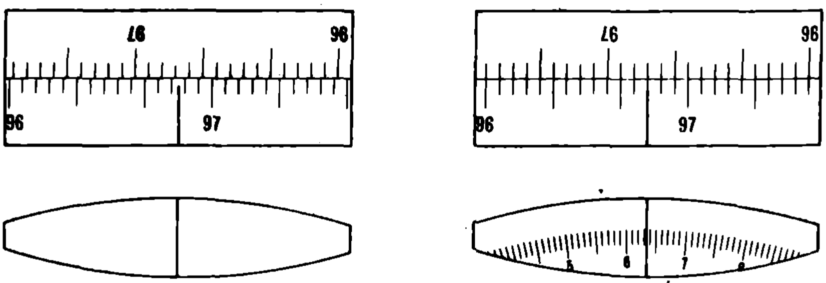
Horizontal Circle

	Circle reading	166° 40′
1	Drum reading	39″·3
2	Drum reading	39·4
	Total reading	166° 41′ 18″·7



Horizontal Circle

	Circle reading	83° 30′
1	Drum reading	45″·6
2	Drum reading	45·5
	Total reading	83° 31′ 31″·1



Vertical Circle

	Circle reading	96° 48′
1	Drum reading	6″·3
2	Drum reading	6·8
	Total reading	96° 48′ 13″·1

The first three diagrams illustrate the Horizontal Circle, the last illustrates the Vertical Circle

Reg. No. 51 D. D. D. 1930—400.

Cards with spider lines
Methylated spirits
Wax
Copal varnish
Watch-maker's eyeglass
Camel hair brush
Scissors.

40. Perfection of pivots.—Ideally the pivots should be two equal co-axial cylinders, but since the bearing on the Y's is fairly short, it will suffice if their sections by the planes of the Y's are two equal circles. Inequality may be tested by carefully reading the striding level and then reversing the pivots in the Y's when it will be revealed by a dislevelment when the striding level is replaced. It is of no importance, in that its only result is a slight dislevelment of the transit axis, the errors due to which cancel with change of face. Any irregular departure from a true circular section is more serious. Large errors may be detected by means of the striding level, the telescope being rotated in altitude with the level in position. If there is any detectable irregularity the pivots require regrinding.

41. Wild Precision Theodolite.—The Wild Precision theodolite is the larger of the two makes of Wild theodolite. The ease with which it can be transported, and the speed with which its readings can be made, are such that, provided it is found to give accuracy equal to that of the 12-inch, the latter will never again be used for triangulation in India. It may even be hoped that the compactness of the design, and the excellent protection given to the circles and working parts, may result in an increase of accuracy: but of this there is so far (1929) no evidence.

After its small size the feature of most interest to the triangulator is the fact that the circles are read by a single reading in a microscope by the side of the main telescope, and so can be made in a small fraction of the time required for walking round and reading the microscopes of an ordinary theodolite. The readings so made involve the coincidence of the images of graduations separated by 180°, and so are free from error due to eccentricity of graduation. The rather complicated optical system by which this is achieved is shown in fig. 7.

42. Reading the Wild.—[See figs. 9 (a) to (d)]. Horizontal and vertical circles are read in the same microscope. By turning the appropriate screw (see fig. 8) clockwise as far as it will go, the horizontal circle is brought into view: the vertical circle is viewed by turning it counter-clockwise. The images of points on the circles separated by 180° are seen together in the microscope, separated by a fine horizontal line. In the case of the horizontal circle the graduations are at four-minute intervals.

To read the horizontal circle the two sets of graduations are brought into coincidence by means of the micrometer head lying on the prolongation of the horizontal axis of the telescope. An index mark appears in the field of view, whose function is to indicate in what part of the field the coincidence is to be made. After making the coincidence this index mark will lie between two numbers indicating degrees. That which lies to the left of the index, and which is read right way up, gives the degrees of the reading. The number of minutes to be recorded is twice the number of four-minute divisions lying between this degree graduation and the degree graduation 180° greater, which is upside down to the right of the index mark. This reading inevitably gives an even number of minutes. The number of seconds remaining (between 0 and 120) is obtained by doubling the reading given by the lower index mark on the circular scale below. In practice, instead of doubling a single reading, it is customary to intersect the mark twice, and so to obtain two independent readings of the seconds, whose sum is taken instead of their mean. This scale is divided into 0 to 60 double seconds, each subdivided to double tenths. Double hundredths can be immediately obtained by estimation, although obviously they are of very little value. The examples in fig. 9 should make this method of reading quite clear.

The vertical circle is read in a similar manner, but its divisions are at eight-minute intervals. The graduations are numbered from 0° to 180° right round the circle, so that an eight-minute interval is numbered as if it were four-minute. Their appearance in the microscope is thus exactly the same as that of the horizontal circle. The readings are made in exactly the same way, the seconds being either doubled or repeated. The required vertical angle is the simple difference between FR and FL (no division by 2). On one face the reading will be between 90° and 135°, and on the other face it will lie between 90° and 45°. The rule is that if FL is more than 90°, the angle is an elevation, and vice versa: i.e., the elevation = FL *minus* FR, with the correct sign.

The following is the method by which the mean reading at two opposite parts of the circle is obtained* (see fig. 7). Light is introduced through the prisms 1 & 2 at the bottom of the theodolite: it is condensed by the lens 3, and passes through the lower parts of the prisms 4, by which it is divided into two halves, on to the graduations on the upper side of the glass circle 5. Here it is reflected by the circle and not by the graduations themselves. It returns through the upper part of 4 and is reflected up the hollow axis of

* See "Revue d'optique" No. 5 of May 1927, for a description by Capitaine Ollivier.

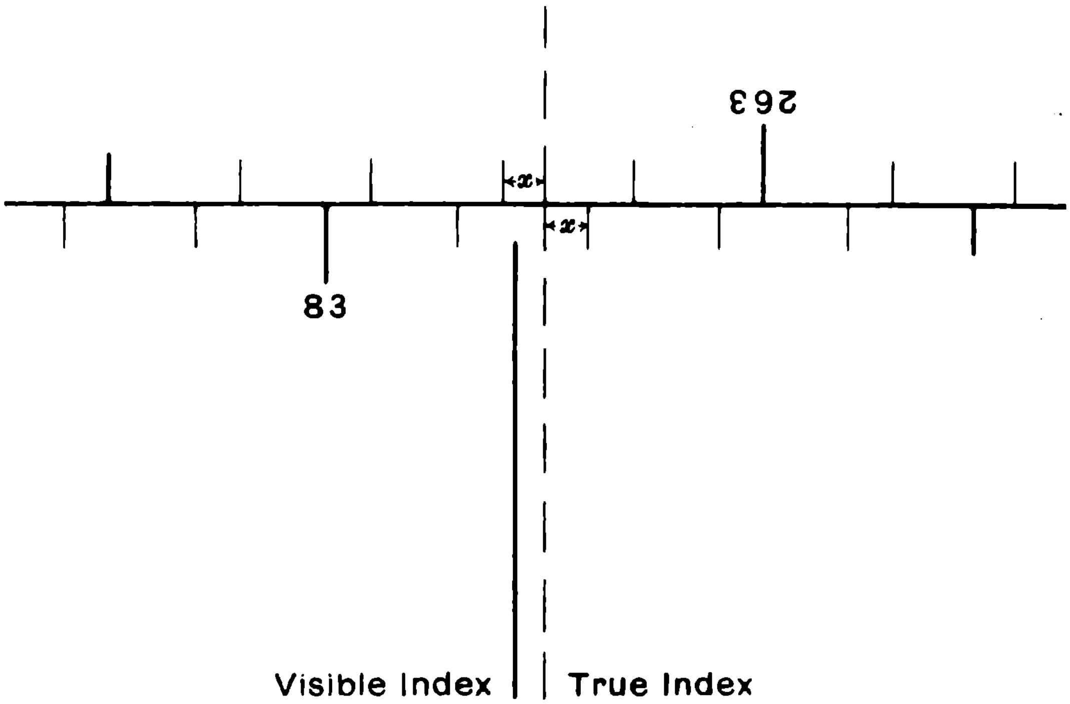


Fig. 10

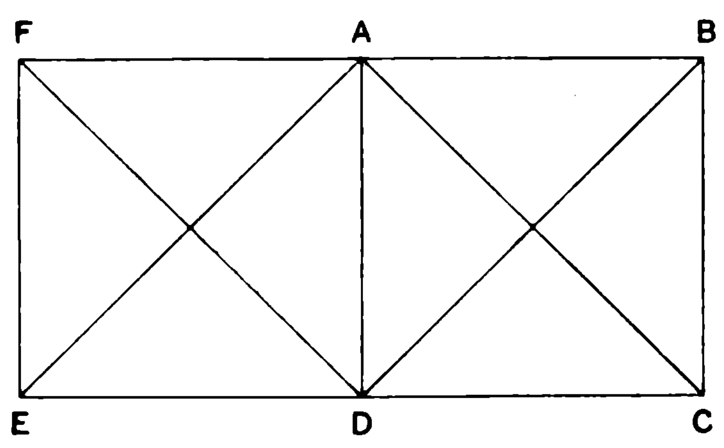


Fig. 11

the theodolite, through the lens 6. The light is then carried to one side by the rhomboid 7, past the prism 8, which is only interposed when the vertical circle is to be read, and through the two parallel plates 9 & 10, each of which receives the light from one side only of the circle 5. These parallel plates can be slightly rotated in opposite directions by the drum 11, and constitute the optical micrometer by which the images are made to coincide in the reading microscope. The light is then reflected to the left by means of the block 12 from which the light emerges in such a manner that the images of the two opposite parts of the circle, as seen in the microscope, appear one above the other separated by a fine horizontal line. They pass through the rhomboid 13, which introduces the image of the graduated disc 14, whose rotation is mechanically connected with that of the drum 11 and the plates 9 & 10. It is from these graduations that the seconds are read. The light is brought to a focus by the lens 15 and reflected into the microscope by the prism 16. The index marks visible in the microscope are cut on the vertical face of 13.

Just as in the 12-inch theodolite (see para 32) the true index mark is not the mark in the auxiliary microscope, nor the zero of the comb in the micrometer, but is the position of the micrometer wires when the drum reads zero, so in the Wild the true index mark is an imaginary point near the visible index mark, but exactly half-way between corresponding graduations in the upper and lower images. And the true reading is the reading of this true index mark on either of the two images, when the seconds micrometer is reading zero.

Thus, in fig. 10 the reading is 83° 08′ *minus* x. The optical micrometer measures the angle x; for, coincidence of the upper and lower graduations is secured by giving them a relative movement $2x$. As has been stated above, this movement is caused by the rotation of the plates 10 & 11, whose rotation is recorded by the disc 14, whose graduations are so spaced that a relative movement of one division of the images of the circles is recorded as one minute. Hence the necessity for doubling or repeating the seconds reading.

The system for the vertical angles is exactly similar. Light is introduced through either of the prisms in the middle of the telescope, is condensed by the lens 3′, and divided and carried to the circle 5′ by the prisms 4′, whence it passes through the lens 6′ and 6″ to the prism 8, and so to the micrometer.

When readings are being made, the lighting of the circles must be as clear and even as possible. This is secured by turning the illuminating prisms. The lighting should never be changed during a measure of an angle, since the apparent position of a graduation depends considerably on the direction from which it is illuminated.

43. Other peculiarities of the Wild.—The Wild theodolite differs from other theodolites in the following respects also:—

(*a*) Setting for zero. Instead of the usual lower plate clamp and slow motion, the Wild can be set to any circle reading by means of a small wheel between two of the foot-screws (see fig. 8). This wheel gives rather a rapid motion, and it is not possible to set correct to the nearest second: nor is it necessary to do so.

(*b*) Bubble. The upper bubble is read by the coincidence of its two ends, on the well-known Zeiss principle. It is centered immediately before every intersection for vertical angles. The lower bubble is graduated in the ordinary way.

(*c*) There is no moving wire in the eye-piece. The ease with which the circle can be read makes one unnecessary: for the circle can be read very nearly as quickly as the drum of a moving wire micrometer.

(*d*) The usual type of diagonal eye-piece is replaced by a prism which can be attached to the 30-magnification eye-piece. Since this end of the telescope cannot be transited, the highest elevation which can be read is about 63°. The use of the zenith prism results in the horizontal inversion of the images seen in the reading microscope.

(*e*) Most of the adjustments are fixed by the makers (see paras 44 & 45). Consequently, it is almost impossible for them to go seriously wrong, provided the instrument receives a thorough inspection at the factory, and has no serious accidents. The absence of field adjustment effects a great saving of time.

(*f*) Unpacking and Packing. The instrument is carried in a metal container instead of in the usual wooden box. It is carried on the back. To open the container the two locking screws at the bottom are undone by the key provided, and the container is carefully lifted off. The tribrach will be found screwed to the body of the theodolite. It is unscrewed, placed on the stand and centered; its three clips are drawn out to receive the foot-screws of the theodolite, and their seatings are dusted. The two clips which hold the theodolite to the base of the container are then opened and the theodolite is lifted out, placed on the tribrach, and the foot-screws secured. The foot-screw which is marked with a brass dot should be placed in that socket of the tribrach which has a similar mark. The theodolite should be lifted by the two standards which carry the trunnions of the telescope.

When packing up, the process is carried out in the reverse order. The eye-piece end of the telescope is depressed until it is almost in contact with the frame, and the telescope is turned in azimuth until the eye-piece is over the marked foot-screw (see fig. 8).

A cotton pad is placed between the telescope and the frame. It is then clamped in azimuth and altitude. When returning the theodolite to the container, the marked foot-screw must coincide with the dot on the outside of the base of the container. The dome must be carefully lowered, with its dot vertically over that of the base, and the clamping screws properly tightened.

Page 35, para 43, sub-para (*g*), lines 3 to 8.

Omit the words "On top of the stand.............capstan screw underneath".

No. 1 dated 8-6-31.

screw underneath.

(*h*) The diaphragm is cut on glass. The vertical wire is partly single and partly double, giving a choice for different types of marks. There is a single horizontal wire.

(*i*) Three eye-pieces are provided of magnification 20, 30 and 40.

44. Adjustment of the Wild.—The only adjustments of the Wild Precision theodolite are:—

(*a*) Centering. As for the 12-inch (para 29). The plumb-bob being fixed to the tribrach, it is not possible to verify its adjustment by rotating the theodolite.

(*b*) Levelling, and adjustment of the levels. As for the 12-inch (para 30), except that the upper level is adjustable by means of the screw seen in fig. 8.

(*c*) Adjustment of the reading microscope for clear vision, by turning the eye-piece. A personal adjustment.

(*d*) Adjustment of the telescope for clear vision of the cross-wires, by turning the eye-piece.

(*e*) Adjustment of the telescope for absence of parallax, as in the 12-inch. The focussing is carried out by turning a has a similar mark. The theodolite should be lifted by the two standards which carry the trunnions of the telescope.

When packing up, the process is carried out in the reverse order. The eye-piece end of the telescope is depressed until it is almost in contact with the frame, and the telescope is turned in azimuth until the eye-piece is over the marked foot-screw (see fig. 8).

A cotton pad is placed between the telescope and the frame. It is then clamped in azimuth and altitude. When returning the theodolite to the container, the marked foot-screw must coincide with the dot on the outside of the base of the container. The dome must be carefully lowered, with its dot vertically over that of the base, and the clamping screws properly tightened.

(*g*) The stand. The Wild tripod is not used for primary triangulation in India. Its place is taken by a heavier wooden stand similar to that of the 12-inch. On top of the stand is placed a heavy circular base plate with three pointed feet, to which the tribrach is screwed. It is important that there should be no play in the three feet, and that the tribrach should be firmly screwed in by means of the capstan screw underneath.

(*h*) The diaphragm is cut on glass. The vertical wire is partly single and partly double, giving a choice for different types of marks. There is a single horizontal wire.

(*i*) Three eye-pieces are provided of magnification 20, 30 and 40.

44. Adjustment of the Wild.—The only adjustments of the Wild Precision theodolite are :—

(*a*) Centering. As for the 12-inch (para 29). The plumb-bob being fixed to the tribrach, it is not possible to verify its adjustment by rotating the theodolite.

(*b*) Levelling, and adjustment of the levels. As for the 12-inch (para 30), except that the upper level is adjustable by means of the screw seen in fig. 8.

(*c*) Adjustment of the reading microscope for clear vision, by turning the eye-piece. A personal adjustment.

(*d*) Adjustment of the telescope for clear vision of the cross-wires, by turning the eye-piece.

(*e*) Adjustment of the telescope for absence of parallax, as in the 12-inch. The focussing is carried out by turning a milled ring on the telescope between the eye-piece and the axis, thus moving the internal focussing lens.

(*f*) Collimation in altitude. As with the 12-inch, the elevation of an object is read on both FL and FR. The circle is then set to the mean reading by means of the altitude slow motion screw. The mark is intersected by means of the bubble-setting screw on the side of the telescope, and the bubble is centered by means of the capstan screws immediately below one end of the bubble.

(*g*) Collimation in azimuth, as for the 12-inch. The diaphragm can be moved by three capstan screws near the eye-piece. To verify this adjustment no special observations are necessary. It is sufficient to note the difference between the face right and face left circle readings of the first signal intersected at each station. The collimation error is half the difference. It should be noted at each station.

(*h*) Determination of the value of one division of the lower plate bubble, as for the 12-inch.

45. Further adjustments of the Wild.—The following adjustments should also be tested, but cannot easily be remedied. The first should be tested and recorded at every station, and the third annually :—

(*a*) Micrometer run. Turn the micrometer drum until the seconds reading is about zero. Secure coincidence by means of the horizontal slow motion screw. With the micrometer drum disturb the coincidence, and then renew it and read the seconds. With the micrometer move the graduations on until another coincidence is secured, and again read the seconds. This reading should differ by 60 seconds from the first. The operation should be repeated several times as above, and also with the first coincidence at about 60 seconds and the second at zero.

(*b*) That the graduations seen in the micrometer are nearly at right angles to the fine line dividing them. For the remedy see para 47(*g*).

(*c*) Transit axis. It is not possible to use a striding level. The horizontal angle between some well elevated object and some low object should be measured on both faces. If the two do not agree, and if the collimation in azimuth is correct, the difference is probably due to a dislevelled transit axis.

46. Stiffness and rigidity.—In addition to all adjustments, theodolites of all kinds, and their stands, must be carefully examined for the slightest stiffness in places where they should move

Page 37, para 46.

After first sub-para *add* the following sub-para :—

In the case of one large Wild theodolite, it has been found that the back-lash, amounting to about 2 seconds, is not taken up by a single rotation of 180 , but that it appears to be taken up in the course of first angle measured, with very serious results to the accuracy of the triangulation. Accurate work has been obtained from this instrument by (a) rotating the telescope about 10 full turns in the direction of swing after any change of face or zero, and (b) observing all the face left measures on any one zero in one continuous swing, followed by the face right measures on the opposite swing. See Geodetic Report Vol. VII, Chap. VI.

No. 2 dated 8-6-31.

... in perfect order.

47. Vertical axis.—It is particularly important that the theodolite should turn easily on its vertical axis. If there is any stiffness the axis must be taken out, cleaned with petrol, and oiled. In the 12-inch theodolite this is not a matter of difficulty, except for the weight of the instrument and the difficulty of supporting the upper part when it is removed. At the extreme bottom of the 12-inch theodolite a dust cap protects the bottom end of the bearing. If this dust cap and one or two capstan screws are removed, the upper part of the theodolite with the vertical axis, can be lifted straight out. The telescope and vertical circle should of course be removed first. In order that the axis may be lifted straight out, the theodolite should be stood on a firm box, or other support near the ground. Great care must be taken not to bend or injure the axis or its bearing, nor to scratch it or let dust settle on it. Both axis and bearing should be washed with petrol, dried, and oiled with a very small quantity of the best clock oil. The axis must be returned vertically into its bearing with great care. It may sometimes happen that the stiffness does not lie in the bearing of the vertical axis but in that of the lower plate, which turns with it. This should also be removed and oiled.

In the 12-inch theodolite the axis and its bearing are conical. The pressure on the conical seating, and consequently the stiffness, can be regulated by means of the cap at the bottom of the axis. If this cap is screwed up, it takes some of the weight. It is then clamped in

46. Stiffness and rigidity.—In addition to all adjustments, theodolites of all kinds, and their stands, must be carefully examined for the slightest stiffness in places where they should move freely, or slackness where they should be rigid. A well-defined point should be repeatedly intersected after a full swing right and after a full swing left, alternately, on the same face. If the results differ systematically, something is slack. This is a much more severe test than the theodolite receives in practice; for, when angles are being measured, the theodolite is swung up to each station in the same direction; and if back-lash is constant, it is largely eliminated. A theodolite would probably be capable of accurate work, if this test showed a difference of as much as five seconds, especially if the difference was constant within a second or two. Nevertheless, all slackness is bad (it may get worse) and every effort should be made to discover and eliminate the cause.

Similarly, all slow motion screws and micrometers should be tested for back-lash by intersecting with them first from one side and then from the other. In practice, intersection will always be made against the spring, but the presence of much back-lash or lost motion will indicate that the instrument is not in perfect order.

47. Vertical axis.—It is particularly important that the theodolite should turn easily on its vertical axis. If there is any stiffness the axis must be taken out, cleaned with petrol, and oiled. In the 12-inch theodolite this is not a matter of difficulty, except for the weight of the instrument and the difficulty of supporting the upper part when it is removed. At the extreme bottom of the 12-inch theodolite a dust cap protects the bottom end of the bearing. If this dust cap and one or two capstan screws are removed, the upper part of the theodolite with the vertical axis, can be lifted straight out. The telescope and vertical circle should of course be removed first. In order that the axis may be lifted straight out, the theodolite should be stood on a firm box, or other support near the ground. Great care must be taken not to bend or injure the axis or its bearing, nor to scratch it or let dust settle on it. Both axis and bearing should be washed with petrol, dried, and oiled with a very small quantity of the best clock oil. The axis must be returned vertically into its bearing with great care. It may sometimes happen that the stiffness does not lie in the bearing of the vertical axis but in that of the lower plate, which turns with it. This should also be removed and oiled.

In the 12-inch theodolite the axis and its bearing are conical. The pressure on the conical seating, and consequently the stiffness, can be regulated by means of the cap at the bottom of the axis. If this cap is screwed up, it takes some of the weight. It is then clamped in position by means of the capstan screw above it. If the axis is raised too much it may become loose in its bearing: this will be shown up by the resulting instability of the instrument's level.

To oil the vertical axis of the Wild theodolite is not so easy, but it can be carried out in the field without much difficulty as follows:—

(*a*) Remove the small prism which introduces light to the horizontal circle, and also its bracket.

(*b*) Remove any one of the three foot-screws thus—

(i) Undo the capstan-headed screw on top of the foot-screw.

(ii) This exposes a small screw inside. Undo it. It is a left handed screw.

(iii) Screw the foot-screw right out.

(*c*) Undo the six small screws spaced round the bottom of the casting of the horizontal circle. The bottom cover of the theodolite may then be pulled out. The prism in the middle comes with it.

(*d*) This exposes the two long prisms which reflect the light on to and from the horizontal circle. They are fastened to a tube which enters, and screws into, the vertical axis. They should be removed by bodily unscrewing them, together with this tube. Some strength may be required. No small screws need be removed.

(*e*) The spring of the horizontal slow motion should be removed by unscrewing the cap at its base. The upper part of the theodolite together with the vertical axis may now be lifted out.

(*f*) The axis and its bearing should be cleaned with petrol, and after drying, oiled with the best clock oil. This should always be carried in the field. Over-oiling should be avoided, on account of the risk of oil getting on to the glass surfaces. With reasonable care this risk is small.

(*g*) The theodolite should then be re-assembled. The two long prisms should not be screwed up with unlimited force as the slight distortion thereby induced in the axis may make it tight in its bearing.

If they are tightened too much or too little, the two sets of graduations of the horizontal circle, seen in the reading telescope, will not appear at right angles to the fine line dividing them. The adjustment need not be very exact, although it is desirable that the scale divisions should be as perpendicular to the dividing line as possible, to avoid difficulty in judging the exact coincidence.

This perpendicularity can be achieved while the prisms are being tightened, by putting the theodolite up in the usual upright position and watching through the microscope as the tightening proceeds. The diffused light from the table is enough to light up the graduations. A large error in this adjustment may cause a small error in the "run" of the micrometer: this should be tested after assembly.

(*h*) After screwing up these prisms, they and the glass plate on the upper side of which the graduations are cut, should be carefully and thoroughly cleaned with a chamois leather.

(*i*) The six small screws round the bottom of the casing of the horizontal circle should not be done up too tight, as their threads are easily stripped.

48. Measurement of angles.—All measures of angles are made in pairs, face left and face right. On face left the telescope is swung left, i.e., from right to left, and on face right it is swung right. Two such measures are known as a "set", and the mean value of a set should be free from any errors of collimation or of dislevelment of the transit axis: it should also be fairly compensated for any small amount of looseness or lack of rigidity in the lower part of the theodolite or in its stand.

Page 39, para 48, second sub-para., lines 8 to 11.

For "In primary triangulation.........a three micrometer", *substitute* the following :—

In primary triangulation three sets are observed on each of 10 zeros in the case of a two-microscope theodolite, and six sets on each of 5 zeros in the case of a three-microscope.

No. 3 dated 8-6-31.

be, their angular separation multiplied by their number should be about equal to 180° in a two-microscope and 60° in a three-microscope *. Thus, if a three-microscope theodolite is being used on 5 zeros, the change between zeros will be about 12° *; with a two-microscope theodolite it will be 18°. For, it is apparent that on a two-microscope theodolite observations on zero 0° cover exactly the same part of the circle as observations on zero 180°. Similarly, zeros 36° and 216° are the same. And on a three-microscope theodolite zeros 0° and 24°, 120° and 144°, and 240° and 264° are respectively identical as also are 0° FL and 60° FR, etc.

* But see page 40. In the case of the three-micrometer theodolite it is customary to change by 132° (viz., $120^\circ + 12^\circ$) in order to obtain greater variety of position of the graduated circle.

perpendicular to the dividing line as possible, to avoid difficulty in judging the exact coincidence.

This perpendicularity can be achieved while the prisms are being tightened, by putting the theodolite up in the usual upright position and watching through the microscope as the tightening proceeds. The diffused light from the table is enough to light up the graduations. A large error in this adjustment may cause a small error in the "run" of the micrometer: this should be tested after assembly.

(*h*) After screwing up these prisms, they and the glass plate on the upper side of which the graduations are cut, should be carefully and thoroughly cleaned with a chamois leather.

(*i*) The six small screws round the bottom of the casing of the horizontal circle should not be done up too tight, as their threads are easily stripped.

48. Measurement of angles.—All measures of angles are made in pairs, face left and face right. On face left the telescope is swung left, i.e., from right to left, and on face right it is swung right. Two such measures are known as a "set", and the mean value of a set should be free from any errors of collimation or of dislevelment of the transit axis: it should also be fairly compensated for any small amount of looseness or lack of rigidity in the lower part of the theodolite or in its stand.

Since the circles of theodolites cannot be perfectly graduated, it is necessary to measure each angle on several different parts of the circle. After one or more sets have been completed, the graduations are moved on a certain number of degrees, and the measures repeated. This process is known as "changing zero", and all measures made between any two changes of zero, are said to be on "zero θ", where θ is the face-left circle reading of one specified station, known as the "zero station". In primary triangulation three sets are observed on each zero, and measures are made on 10 zeros in the case of a two-microscope theodolite and 5 zeros in the case of a three-micrometer. Whatever the number of zeros may be, their angular separation multiplied by their number should be about equal to 180° in a two-microscope and 60° in a three-microscope *. Thus, if a three-microscope theodolite is being used on 5 zeros, the change between zeros will be about 12° *; with a two-microscope theodolite it will be 18°. For, it is apparent that on a two-microscope theodolite observations on zero 0° cover exactly the same part of the circle as observations on zero 180°. Similarly, zeros 36° and 216° are the same. And on a three-microscope theodolite zeros 0° and 24°, 120° and 144°, and 240° and 264° are respectively identical as also are 0° FL and 60° FR, etc.

* But see page 40. In the case of the three-micrometer theodolite it is customary to change by 132° (viz., 120° + 12°) in order to obtain greater variety of position of the graduated circle.

In order to cancel out any error arising from imperfection in the run of the micrometers, the zero settings should not be whole numbers of degrees, but should be whole degrees *plus* a varying number of minutes, the minutes of the different settings progressing evenly through one graduation interval of the circle. It is not possible to make zero settings with any great accuracy (especially on the 12-inch theodolite), but they should be made carefully with errors of not more than 30 seconds. If all zero settings were whole degrees, an error due to micrometer run would be unchanged on all zeros, and the run would have to be adjusted with almost impossible accuracy. By this system the error caused by inaccurate run is reduced by 80°/o, and an error of one or even two seconds per 10 minute run has negligible results.

The zeros to be normally used are :—

12-inch 2-microscope			12-inch 3-microscope			Wild.					
						Actual setting			Apparent setting		
°	′	″	°	′	″	°	′	″	°	′	″
0	00	00	0	00	00	0	00	00	0	00	00
18	00	30	132	01	00	18	00	24	18	00	12
36	01	00	264	02	00	36	00	48	36	00	24
54	01	30	36	03	00	54	01	12	54	00	36
72	02	00	168	04	00	72	01	36	72	00	48
90	02	30				90	02	00	90	02	00
108	03	00				108	02	24	108	02	12
126	03	30				126	02	48	126	02	24
144	04	00				144	03	12	144	02	36
162	04	30				162	03	36	162	02	48

The labour involved in a change of zero is small. If it is found that the difference between measures on different zeros are much larger than is to be expected from the accordance of different measures on the same zero, the number of zeros should be increased, and fewer measures made on each. Thus, instead of 3 sets on 10 zeros, there may be substituted 2 sets on 15 zeros, or even a single set on each of 30 zeros. On the other hand, the number of zeros should not be increased unnecessarily, as apart from a little saving of time in reading and recording, the smaller number of zeros, with two or three sets on each, provides the observer with a good criterion of the steadiness of his observations, and of the absence of changes in lateral refraction. If zero is changed between every set, avoidable errors may be attributed to graduation error, and so passed unnoticed.

In the 12-inch theodolite the eye-piece is provided with a moving wire. Intersection with this wire, and readings on its micrometer drum, can be repeated in a small fraction of the time required to turn the telescope and read the circle micrometers. Consequently it is customary to make several intersections with the moving wire, at each pointing of the telescope. This is especially useful when the atmosphere is unsteady and intersections are doubtful. The rules for the use of the eye-piece micrometer are:—

(1) That all observations will be made within 15 divisions of the zero position.

(2) That at least 3 intersections will be made, and that if their reading ranges through more than three seconds, repetitions will be made until the number of readings equals the number of seconds in their total range. In very flat country it may be necessary to relax this rule.

In the Wild theodolite there is no moving wire, but the circle is read with extreme ease. Two readings will always be made (see para 42), and if they differ by more than two seconds (one double second) repetitions will be made as above. It will generally be convenient to make repetitions in pairs.

The detailed procedure is as follows:—Suppose the observer is at A (see fig. 11) and that stations B, C, D, E and F are visible. Either station B or F will be selected as zero station: the right-hand station is the more convenient, as the zero setting is most conveniently made for face left, swing left. If F is selected, the first swing will be to the left, so the theodolite will be set on FL, and the graduations will be so placed that the reading is 0° when the telescope is directed on F. The telescope is then pointed about 30° to the right of F and gently swung up to it, care being taken that the telescope is not swung beyond it further than can be brought back by the slow motion screw. If it is overswung it must be swung right round the circle until it is just short of the mark. The mark is then intersected with the slow motion screw, the last motion of the screw being against the spring. If the mark is overshot, the screw must be turned well back and the intersection repeated. The final intersection is then made with the moving wire (if any), care again being taken that the last movement is against the spring. The necessary intersections having been made and recorded, the circle micrometers are read to the nearest tenth of a second. In the Wild theodolite, the circle is read at each intersection (as explained above). The upper circle is then unclamped, the moving wire is set to approximately zero, the telescope is swung on to E and the process repeated, until the reading of B has been completed. Face is then changed to FR, the telescope being swung round in such a direction that it comes up to B from the left, and the round is repeated from left to right. This completes one set. The next set may conveniently be taken first on FR, swing right, beginning at B. The first set having been completed by the intersection of F, the telescope must be swung round to B by a continuous right handed movement.

With regard to the direction of swing of the telescope, the fundamental principles are:— (1) That measures of every angle must be made in pairs of swing right and swing left. (2) That, apart from this, the motion must be continuous in one direction without reversal. (3) That, after a reversal of the direction of movement, the telescope should be swung at least 30° in the new direction before any intersection is made.

If, at any intersection after the first of a round, the telescope overshoots the mark beyond the range of the slow motion screw, it will not suffice to swing round the circle, and then to re-intersect. The telescope must be swung round the circle to the preceding station, that preceding station must be re-read and recorded, and the round continued from that point. If, as is bound to be the case, the second reading of that station is not identical with the first, the first reading should be used to deduce the angle between it and the stations preceding it, and the second reading should be used with those of succeeding stations.

In the above example the stations F to B have all been on one side of the station A, leaving an angle of about 180° whose measure is not required. At the central station of a polygon where all the angles are required, or at any station where the unwanted angle is less then 90° the procedure will be a little different. Starting at F the telescope will intersect the stations in turn through E to B, and will then be swung on in the same direction until F is re-intersected and re-read. If the second reading agrees with the first within such small limits as may reasonably be attributed to reading error and to lateral refraction, the first measure will be used in conjunction with E, and the second with B. The difference between the two readings will not be distributed through the round. If the difference is exceptionally large, it must be attributed to movement of the graduated limb, and the round must be rejected and repeated. After a swing left has been satisfactorily completed, face should be changed, the telescope redirected on F, and the round repeated in the opposite direction.

When the appointed number of sets on the first zero have been completed, zero is changed, and measures on the next zero are made then changed to FR, the telescope being swung round in such a direction that it comes up to B from the left, and the round is repeated from left to right. This completes one set. The next set may conveniently be taken first on FR, swing right, beginning at B. The first set having been completed by the intersection of F, the telescope must be swung round to B by a continuous right handed movement.

With regard to the direction of swing of the telescope, the fundamental principles are:— (1) That measures of every angle must be made in pairs of swing right and swing left. (2) That, apart from this, the motion must be continuous in one direction without reversal. (3) That, after a reversal of the direction of movement, the telescope should be swung at least 30° in the new direction before any intersection is made.

If, at any intersection after the first of a round, the telescope overshoots the mark beyond the range of the slow motion screw, it will not suffice to swing round the circle, and then to re-intersect. The telescope must be swung round the circle to the preceding station, that preceding station must be re-read and recorded, and the round continued from that point. If, as is bound to be the case, the second reading of that station is not identical with the first, the first reading should be used to deduce the angle between it and the stations preceding it, and the second reading should be used with those of succeeding stations.

In the above example the stations F to B have all been on one side of the station A, leaving an angle of about 180° whose measure is not required. At the central station of a polygon where all the angles are required, or at any station where the unwanted angle is less then 90° the procedure will be a little different. Starting at F the telescope will intersect the stations in turn through E to B, and will then be swung on in the same direction until F is re-intersected and re-read. If the second reading agrees with the first within such small limits as may reasonably be attributed to reading error and to

Page 42, para 48.

Insert the following sub-para above the last two lines of the page:—

When the theodolite in use is suspected of any tendency to measure all angles too small, on account of back-lash in the clamps, footscrews or stand, it will be better to complete all the face left measures on any one zero in one continuous swing, before changing face. By this means only the first angle measured will be likely to be affected by back-lash, and any progressive slipping will be shown up by the changes in the circle reading. See para 46, and Geodetic Report Vol. VII, Chap VI.

No. 4 dated 8-6-31.

in the same way. It will generally be convenient to begin each zero on FL.

49. Broken rounds.—It will often happen that all the signals are not visible. In such a case the observer should proceed with the intersection of such stations as are visible. The absence of one station will result in his obtaining measures of the sum of two of the required angles, instead of their separate values. As soon as the missing station becomes visible, he must measure the angle between it and either of its neighbours, thus directly measuring one of the missing angles, the value of the other being obtained by subtraction. These supplementary measures must be entered in the angle book in the order in which they are made: they should not be incorporated with the readings of the original round, of which they should have formed a part. Although often unavoidable, this procedure is not a desirable one. It wastes time, it confuses the record, and it provides indirect measures of less weight than ordinary direct measures. The observer must do his best to avoid it, in so far as it can be prevented by good training and organisation of his signal squads. But if a signal is not showing when required, it will seldom pay to wait for it more than one minute before proceeding to the next intersection. Under some conditions (e.g., when the sun is rather occasionally showing through clouds) it is best to concentrate on the measurement of one angle at a time.

It is to be remembered that the two intersections, which constitute the measure of an angle, must not be separated by an interval of several minutes, but must follow each other quickly. Temperature changes in the stand and instrument are likely to cause movement of the graduated circle during the delay.

The zero setting for a broken round is obtained by adding the required reading of the zero station to an approximate value of the angle between that station and the first station included in the broken round.

50. Time of Observation.—The normal procedure is to observe about half the angles by day and half by night, but this is not a rule to be rigorously enforced, and, if the observer thinks it desirable, the angles may be measured wholly by day or wholly by night, except for grazing rays (see below). If the rays are long, and the atmosphere a little hazy, lamps may be invisible, while helios may show up fairly well. On the other hand in clear but cloudy weather the lamps will show better than the helios. If the rays pass at all close to the ground, the marks may be so unsteady during the middle of the day that observations at that time are worthless, however clearly the helios may be showing. With such rays the best time is in the evening and at night. In flat ground morning observations are seldom possible.

A ray which grazes past the side of a hill, and so is suspected of suffering from lateral refraction, should be observed half by day and half by night. According as the hill is hotter or colder than the surrounding air, so is the sign of the lateral refraction likely to be reversed. Consequently the mean of day and night observations has a chance of being considerably better than either, although no reliance can be placed on this. Of more consequence is the fact that the error due to lateral refraction is unlikely to be greater than the difference between the day and night observations, so that its presence will be revealed*; although there is no certainty of this either, unless the hill is fairly free from jungle and well exposed to the sun. To sum up, grazing rays must be avoided. If there is any suspicion of a graze, observations must be made both by day and by night. If the daily mean differs materially from the nightly mean, it is obvious that the graze is affecting the ray, whereas if day and night values do not differ much, and if the surface of the hill is well exposed to the sun, it may reasonably be hoped that the graze will have done no harm. (See Appendix I).

51. Miscellaneous Instructions for Observers:—

(*a*) In primary triangulation an observatory tent will always be used.

(*b*) When a 12-inch theodolite is being turned in azimuth it should be held by the knobs provided on the ring which guards the horizontal circle. A Wild theodolite should be turned by the standards. A theodolite should not be turned by the telescope. The movement should always be slow and even. Besides the risk of moving the lower plate, rapid movement will probably result in the mark being overshot.

(*c*) The theodolite should be turned a few revolutions in altitude and azimuth before starting work every morning.

(*d*) The theodolite must be kept clean, and frequently dusted. Dirt on the outside may do no harm where it is, but it is likely to get inside. The theodolite should always be dusted with a small brush before work, the painted surfaces first and the bright surfaces later. In particular the pivots and the bearings (in the 12-inch) must be kept clean. Lenses should be dusted lightly with the brush, but rubbed as little as possible. If greasy marks have to be removed, the lens should be rubbed with very soft paper slightly damped. The component parts of a compound lens should never be

* Unless the graduation error of the instrument is known, or is negligible, it is useless to compare observations on different zeros for this purpose.

separated unless absolutely necessary. If it has to be done, they must be clearly marked beforehand so that they can be replaced in exactly the same relative position, with the same faces touching and without relative rotation.

(*e*) A 12-inch theodolite should be lifted by its tribrach or foot-screws by two men. A Wild theodolite is lifted by its standards.

(*f*) Intersections should be made close to but apart from the horizontal wire. When the diaphragm is provided with a vertical wire which is partly single and partly double, the double wire is generally preferable for use with luminous signals.

(*g*) When eye-pieces of different magnification are provided, the highest power should generally be used for luminous signals, even when bad atmospheric conditions cause the helio to spread; the large magnification may help the observer to select the right point of intersection; low magnification merely hides the difficulty without remedying it. A lower power may be more suitable for intersected points.

(*h*) If a helio appears so bright as to dazzle the eye, it should be dimmed by holding a screen of muslin in front of, but not touching, the object-glass. Screens of single, double and triple muslin may be kept for the purpose.

(*i*) Except in stormy weather a 12-inch theodolite should be left undisturbed on its stand throughout the observations at any one station. It should be protected by a suitable cloth or flannel cover. On the other hand a Wild theodolite should be taken down and returned to its container, whenever observations are interrupted.

(*j*) It is essential that cooking tents and fires should be well removed from the station. Hot air rising from them will cause serious lateral refraction. It is desirable that they should be kept well away from the line of any ray, and that all fires should be reduced to a minimum while observations are in progress.

(*k*) The observer must be careful not to place his feet on the pillar on which the theodolite stands.

(*l*) The height of the theodolite must suit the height of the observer. This must be verified before leaving headquarters, where the legs of the stand can be cut down if necessary, or a fresh stand can be made.

(*m*) Except when the signal appears to be moving from side to side, the observer should not dwell on his intersection.

He should train himself to make his intersections carefully but rapidly, and to proceed to the reading of the circle as soon as he is satisfied. He should occasionally watch the light for half a minute or so, without moving the wire, to detect the presence of slowly changing lateral refraction.

(*n*) The variations of vertical refraction are not fully understood, and further research can only be based on fresh data. Observers should make use of any opportunities of supplying fresh data, which may present themselves. Suitable data are a series of careful vertical angles to different points lying in similar directions, but at different distances and elevations, taken throughout the day at about hourly intervals, together with hourly temperature readings (at both ends of the ray if possible), and a measure of the minimum temperature during the preceding night.

(*o*) When horizontal angles are being observed the vertical axis must not be tightly clamped. It should be lightly clamped, just sufficiently to enable the vertical slow motion screw to move the telescope.

(*p*) When measuring horizontal angles the mark should be brought close to the horizontal wire with the vertical slow motion screw. The last turn of this screw should always be in a screwing up direction, since in some instruments it has been found that the horizontal pointing varies considerably according to the direction of the last movement of this screw.

52. Intersected points.—Intersected points must be fixed with the accuracy demanded by the purpose which they are intended to fulfil. It is not ordinarily the duty of the geodetic triangulator to fix intersected points: this can generally be done more economically by another agency. It will generally suffice to observe them FL and FR on a single zero. Similarly a single pair of FL and FR vertical angles will also suffice. Vertical angles should be observed during the hours of minimum refraction, but if these hours are wholly occupied by the observations to stations, or if the points are then invisible, the intersected points may be observed at an earlier or later hour, provided that one or two stations are re-observed at the same time.

The horizontal angles should be observed as a complete round, beginning and closing on one of the stations of the triangulation. If many points are being observed it is prudent to close the round after every ten points, as a precaution against any movement of the theodolite which would require the repetition of the whole round.

53. Vertical angles.—Vertical angles must be observed to all stations and intersected points. On account of the uncertainty of atmospheric refraction in the vertical plane it is impossible to measure vertical angles with the same precision as horizontal angles, so that fewer measures are made. For instance it is unnecessary to change zero on the vertical circle, and no allowance for such change is made in the design of many theodolites, although in some 12-inch theodolites it is possible.

Unless there are good reasons for the contrary (such as perpetual invisibility of marks during the day), all vertical angles must be observed at the time of "minimum refraction", i.e., between 13.00 and 16.00 hours. Frequent neglect of this rule practically amounts to an omission to measure the heights of the stations. On the other hand it may be recognised that the heights of primary stations are of less importance than their latitudes and longitudes, and that the omission of the vertical angles on a certain number of rays does not break the continuity of the determination of heights throughout the series. Nevertheless, the omission of any vertical angles does destroy the closures of height round two triangles, thus sacrificing a check on the work, and an important measure of its accuracy. As a general rule, work at no station should be prolonged for more than one extra day solely with a view to obtaining or completing the vertical angles. If circumstances enforce the omission of some vertical angles, it must be ensured that there is at least some continuous line of stations from one end of the series to the other, between which reciprocal observations have been made at the time of minimum refraction.

The virtue of observing at the time of minimum refraction does not lie in the smallness of the refraction at that time, but in its exceptional constancy. If reciprocal angles are observed between two stations, and if the atmospheric conditions are the same at each end* and on the different days of observation, the error due to refraction will cancel, irrespective of whether its amount can be correctly calculated or not. At the times of minimum refraction these conditions are fairly closely satisfied, especially in rays from one hill to another, passing well clear of the ground. At night and in the early morning, these conditions are not satisfied and observations at those hours are of no use. The fact that the signals may appear steady at night and in rapid irregular movement by day must not delude the observer into supposing that the former time is the best for vertical angles.

If it is impossible to observe between 13 and 16 hours, observations should be made as near these hours as possible. If frequent

*If the stations are of different heights, conditions will not normally be the same at each end of the ray. But this normal difference is calculated and allowed for in the computations.

difficulty is being experienced, so that there is risk of its being impossible to obtain even a single chain of stations with good reciprocal vertical angles, the best chance of good results will be secured if vertical angles (and temperatures) are measured hourly from dawn to as late an hour as possible, the temperature readings being continued up to 14.00 or 15.00 hours. Late midday observations to one or two stations will help the reduction of earlier observations to other stations, provided the stations which are observed at midday are also observed at the earlier hour.

The mention of the above expedients must not deter the observer from making all possible efforts to observe at the proper time. The only valid excuse for not observing between 13 and 16 hours is that the stations were invisible.

All vertical angles to stations should be measured on at least two different days. Observations will be in pairs FL and FR. A measure of a vertical angle on one face only is worthless. Unless exceptional divergences are shown, four such pairs will suffice, two each day. With the Wild theodolite, intersections and seconds readings are made in pairs in the same way as for horizontal angles. With the 12-inch theodolite a single reading will generally suffice, but if the signals are very unsteady, the moving wire micrometer may be rotated 90°, and brought into use. It should be remembered that this will upset the adjustments for collimation and verticality of the wires, and possibly that for parallax. With the 12-inch theodolite the upper bubble must always be read and recorded; with the Wild it is brought to the centre of its run immediately before each intersection.

Some levels are graduated from the centre outwards, and others consecutively from one end to the other. Different rules for correcting the observed angles for dislevelment are necessary in the two cases. For levels graduated from the centre outwards the formula is as follows:—

Level correction $= \dfrac{\Sigma(\text{O}) - \Sigma(\text{E})}{n} \times d$, where $\Sigma(\text{O})$ and $\Sigma(\text{E})$ are the sums of the readings of the object and eye-ends respectively, d is the value of one division of the level scale in seconds, and n is the total number of level readings counting each pair of eye-end and object-end observations as two readings. This correction is to be applied with its resulting sign to altitudes and with reversed sign to depressions and zenith distances.

For levels graduated from one end to the other, the formula is as follows:—

Level correction $= \dfrac{\pm \Sigma(\text{O}_\text{R} + \text{E}_\text{R}) \mp \Sigma(\text{O}_\text{L} + \text{E}_\text{L})}{n} \times d$, where

$\Sigma(O_R + E_R)$ and $\Sigma(O_L + E_L)$ are the sums of the object and eye-end readings on face right and face left respectively, and d and n are as above. The $\frac{\text{upper}}{\text{lower}}$ sign gives the correction with its resulting sign which is to be applied to $\frac{\text{depressions and zenith distances}}{\text{altitudes}}$ if the graduation reads from the object-end towards the eye-end when the instrument is face right, and to $\frac{\text{altitudes}}{\text{depressions and zenith distances}}$ if the graduation reads in the reverse order on the same face.

The bubble correction must be applied to each mean of FL and FR.

The system of the graduations of the vertical circle varies in different instruments. In 12-inch theodolites the circle is graduated from 0° to 360° in such a manner that elevations are directly read on FR, and 180° *minus* elevations on FL. The observer must examine his theodolite and discover the rule by which the mean value is to be obtained, and how elevations are to be distinguished from depressions.

The temperature, pressure, and time of day must invariably be recorded, together with a few remarks about the state of the air, for example whether the preceding few hours have been sunny or cloudy, whether the signals are still or jumping, whether there has been rain or thunderstorms, or if they appear to be threatening. Of the above, the time of day is the most important.

The height of the instrument above the upper mark-stone must be recorded, as must also the height of the helio or signal left at the station. All measurements are to refer to the upper mark-stone.

54. Illumination at night.—At night the Wild theodolite is illuminated by electrical attachments. The lighting prism of the horizontal circle and either of the prisms on the telescope are unscrewed, and the two lamp holders screwed into their places. That marked 1 is for the horizontal circle, and 2 for the vertical circle. For screwing in 2 the telescope must be tilted until the eye-piece nearly touches the supports, to enable the holder to clear the vertical clamp while it is being screwed in. The vertical circle light also illuminates the cross-wires. For reading the bubble a hand torch is provided, lit by the same battery as the lamps used for reading the circles, but it is generally more convenient to disconnect this torch and to use an ordinary torch with batteries self-contained.

Increased accuracy can probably be obtained by electric illumination of the circles by day as well as by night.

With the 12-inch theodolite the circles are illuminated by means of an electric torch directed on to the white reflectors above the circles. This light must be directed radially from the front of the observer's body. It must not be directed transversally, or shadows will cause an apparent displacement of the graduations. The cross-wires are lighted through the transit axis by an electric lamp.

55. Recording.—It is essential that the record should be kept in a clear legible hand with practically no erasures. Unavoidable corrections must be neatly made in such a manner that the original entry is still legible, but corrections should very seldom occur in the early columns. If for any reason, considerable confusion arises in the angle book, it is permissible to make a fair copy **at the time**, but this copy must be labelled as such on every page, and **the original must** on no account be destroyed or suppressed. The circumstances necessitating the preparation of a fair copy should be noted on both fair and original, and signed by the observer.

Different forms of angle book are provided for the recording of horizontal and vertical angles, and for the 12-inch and Wild theodolites (see figs. 12 to 15). If all or most of the stations are regularly showing, the recorder **may** enter their names in due order, a short time in advance of the observations. But if the observations are being made in irregular order the observer must call out the name of the station before the recorder can enter it.

The face and zero must be carefully recorded, and responsibility lies with the recorder to notice that the face has been called correctly, and that changes of face and zero are made in the correct order. The observer will explain to the recorder any departures from the regular order which he may find necessary.

All readings of the micrometers and levels must be repeated by the recorder as he writes them down, and the computation of the means and of the correction for the eye-piece micrometer must proceed concurrently with the recording, so that irregular values can at once be detected and repeated before the next change of zero. The recorder must immediately draw the observer's attention to any discrepancies between eye-piece micrometer readings, or angular measures, which are beyond the normal limits laid down, and he should notice any gross error in the reading of the degrees and minutes. It is desirable that the abstract should be kept up with the observations. This can be done by a practiced recorder, who can thus point out any wide values which are obtained.

The means of pairs of FL and FR will not be taken out except in the case of horizontal angles to heavenly bodies, of vertical angles to all objects, or when a very large error of collimation or

1 Trian.

Survey of India

Fig. 12

NO. 15 PARTY, Triangulation, TRIGONOMETRICAL SURVEY, SEASON 1930-31

Angles taken at Taungnio H.S. *Station with* 12 *Inch Theodolite No.* V *Date* 1 - 11 - 1930.

Object	Face and Zero	Microscope Readings A	Microscope Readings B	Microscope Readings C	Mean = D	Micrometer 1ᵈ = 1″.147 Mean of readings	Micrometer Value in arc = E	Corrected Mean = D ± E +on FL, − on FR.	Angles	Micrometer Readings and Remarks Lamps 20.00, T = 78°.2 d	d	d	d	d	d
Ayodaung H.S.	L/O	359 59 49 2	69 7	67 9	62 3	+ 4 6	+ 5 3	0 00 07 6		+4·5	+4·8	+4·5			
Pegu S.	"	87 10 37 9	47 6	48 3	44 6	+ 0 4	+ 0 5	87 10 45 1	87 10 37 5	+0·7	+0·4	+0·2			
Mako S.	"	109 51 10 6	15 7	25 0	17 1	+ 0 4	+ 0 5	109 51 17 6	22 40 32 5	+0·4	+0·5	+0·3			
Chanakpo H.S.	"	155 23 28 6	28 7	46 5	34 6	+ 2 3	+ 2 6	155 23 37 2	45 32 19 6	+2·3	+2·6	+2·1			
Chanakpo H.S.	R/O	335 23 27 3	41 1	41 0	36 5	+ 2 4	+ 2 7	335 23 33 8	45 32 15 4	+2·8	+2·0	+2·4			
Mako S.	"	289 51 07 6	24 7	26 3	19 5	+ 1 0	+ 1 1	289 51 18 4	22 40 35 3	+0·9	+1·0	+1·1			
Pegu S.	"	267 10 33 4	51 0	50 0	44 8	+ 1 5	+ 1 7	267 10 43 1	87 10 35 4	+1·4	+1·4	+1·7			
Ayodaung H.S.	"	180 00 04 0	15 0	24 1	14 4	+ 5 8	+ 6 7	180 00 07 7		+5·6	+5·8	+5·9			
Chanakpo H.S.	R/O	335 23 22 1	35 3	35 0	30 8	− 1 9	− 2 2	335 23 33 0	45 32 14 6	−2·5	−2·1	−1·1	−2·0		
Mako S.	"	289 51 07 0	23 9	25 0	18 6	+ 0 2	+ 0 2	289 51 18 4	22 40 35 5	−0·1	+0·6	+0·0			
Pegu S.	"	267 10 33 6	51 7	50 6	45 3	+ 2 1	+ 2 4	267 10 42 9	87 10 36 2	+2·0	+2·0	+2·3			
Ayodaung H.S.	"	180 00 01 0	11 8	23 4	12 1	+ 4 7	+ 5 4	180 00 06 7		+4·3	+4·9	+5·0			
Ayodaung H.S.	L/O	359 59 55 3	62 9	64 2	60 8	+ 6 1	+ 7 0	0 00 07 8		+6·0	+6·0	+6·2			
Pegu S.	"	87 10 36 6	43 4	45 0	41 7	+ 3 4	+ 3 9	87 10 45 6	87 10 37 8	+3·3	+3·4	+3·5			
Mako S.	"	109 51 08 6	13 5	22 7	14 9	+ 4 5	+ 5 2	109 51 20 1	22 40 34 5	+4·5	+4·3	+4·8			
Chanakpo H.S.	"	155 23 23 8	24 2	41 4	29 8	+ 6 9	+ 7 9	155 23 37 7	45 32 17 6	+6·6	+7·1	+6·9			

Registered by A.B.C. *Examined by* G.H.I.

Observed by D.E.F. *Compared by* D.E.F. *and* G.H.I.

5 Trian.

Fig. 13

Survey of India

NO. 15 PARTY (Triangulation) SEASON 1930 – 31

Vertical Angles taken at Taungnio H.S. *Station with* 12 *Inch Theodolite No* V, 1.11. 1930

Object	Face	Time of Observation H. M.	Micrometer Readings G ° ′ ″	Micrometer Readings H ° ′ ″	Level No. 1 Divn. of Scale = 1″.24 Object End d	Eye End d	Mean Vertical Angles One Face ° ′ ″	Both Faces ° ′ ″	General Mean ° ′ ″	Height of Eye Feet	Height of Object Feet	Remarks
Ayodaung H.S.	L	13 30	179 34 31 7	43 2	29 0	29 0	179 34 37 5	I		5 5	2 5	13·30
	R	13 42	0 26 15 1	23 0	34 6	31 1	0 26 19 1	+ 1 1 E 0 25 50 8	E 0 25 51 9			T = 87°·2
	R	13 57	0 26 13 5	21 0	31 8	26 0	0 26 17 3	E 0 25 51 7				β = 29″·2
	L	14 10	179 34 27 0	40 5	26 2	31 5	179 34 33 8	+ 0 2 I				Helio rather unsteady.
								I				Fine with bright sun
								I				all day.
Pegu S.	L	13 34	180 26 22 7	37 6	26 3	31 2	180 26 30 3	I		5 5	2 5	14·20
	R	13 47	359 34 22 4	30 1	31 3	25 6	359 34 26 3	− 0 3 D 0 26 02 0	D 0 26 02 1			T = 87°·4
	R	14 03	359 34 24 6	33 6	27 2	30 7	359 34 29 1	D 0 25 59 7				β = 29″·2
	L	14 15	180 26 28 3	28 6	21 7	27 2	180 26 28 5	+ 2 8 I				
								I				
								I				
								I				
								I				
								I				
								I				
								I				

Registered by A.B.C. *Examined by* D.E.F. *Observed by* G.H.I.

Fig. 14

24 Trian. **Survey of India** Page 5

Angles taken at Rājkot H.S. *with Large Wild Theodolite No.* 111 *Dated* 1.11.30

Object	Face and Zero	Horizontal Angles: Actual Reading °	′	″*	Reduced Reading °	′	″	Mean of faces °	′	″	Angle °	′	″	Remarks
	L/O	00	00	03·2	00	00	06·6							Helios
Sultānpur H.S.				03·4										T = 82°·3
	L	31	16	19·7	31	16	39·2				31	16	32·6	08·10
Khānpur H.S.				19·5										
	L	78	40	21·3	78	40	43·1				47	24	03·9	
Adilabād H.S.				21·8										
	L	132	46	32·3	132	47	05·4				54	06	22·3	
Bijnor H.S.				33·4										
				32·5										
				32·6										
	L	00	00	04·1	00	00	07·8							
Sultānpur H.S.				03·7										
	L	31	16	19·0	31	16	38·5				31	16	30·7	
Khānpur H.S.				19·5										
	L	78	40	22·4	78	40	44·4				47	24	05·9	
Adilabād H.S.				22·0										
	L	132	46	31·8	132	47	04·0				54	06	19·6	
Bijnor H.S.				32·2										
	L	00	00	03·6	00	00	06·9							
Sultānpur H.S.				03·3										
	L	31	16	18·9	31	16	38·2				31	16	31·3	
Khānpur H.S.				19·3										
	L	78	40	21·4	78	40	43·0				47	24	04·8	
Adilabād H.S.				21·6										
	L	132	46	31·9	132	47	04·1				54	06	21·1	
Bijnor H.S.				32·2										

* Divisions of micrometer read 0·2″, reduced reading is the sum of two actual readings.

Observed by A.B.C. *Recorded by* D.E.F. *Checked by* G.H.I.

Fig. 15

Angles taken at Rājkot H.S. *with Large Wild Theodolite No.* 111 *Dated* 1 : 11 : 1930

Object	Face	Vertical Angles												Time
		Actual Reading			Final Reading			Difference			Zenith Distance			
		°	′	″*	°	′	″	°	′	″	°	′	″	h. m. s.
Sultānpur H.S.	L	90	08	50·5	90	09	41·2							13 – 30
				50·7				E 0	19	20·2				T = 87°·2
"	R	89	50	10·2	89	50	21·0							β = 29°·2
				10·8										Helios rather
"	R	89	50	09·3	89	50	19·0							unsteady.
				09·7				E 0	19	23·0				Fine weather,
"	L	90	08	50·8	90	09	42·0							bright sun
				51·2										all day.
Khānpur H.S.	L	89	28	37·3	89	29	13·8							
				36·5				D 1	01	33·1				
"	R	90	30	23·8	90	30	46·9							
				23·1										
"	R	90	30	19·9	90	30	40·5							
				20·6				D 1	01	30·2				
"	L	89	28	35·1	89	29	10·3							
				35·2										
Adilabād H.S.	L	89	30	46·1	89	31	31·7							
				45·6				D 0	56	56·6				
"	R	90	28	14·0	90	28	28·3							
				14·3										
"	R	90	28	13·9	90	28	27·4							
				13·5				D 0	56	53·6				
"	L	89	30	47·1	89	31	33·8							
				46·7										
Bijnor H.S.	L	89	18	25·5	89	18	50·5							
				25·0				D 1	22	18·9				
"	R	90	40	35·1	90	41	09·4							
				34·3										
"	R	90	40	32·1	90	41	05·2							14 – 10
				33·1				D 1	22	18·4				T = 87°·4
"	L	89	18	23·2	89	18	46·8							β = 29°·2
				23·6										

At Station Rājkot { *Height of Helio* 2′ 6¼″ ; *Height of instrument* 5′ 1″ }

* Divisions of micrometer read 0·2″, so difference of face R & L readings come out in 0·1″ units without division by 2.

Observed by A.B.C. *Recorded by* D.E.F. *Checked by* G.H.I.

dislevelment of the transit axis causes a noticeable discrepancy in the horizontal angles to terrestrial objects.

With the 12-inch theodolite the eye-piece micrometer readings must be meaned, multiplied by the value of one division of the drum, and also by the secant of the angle of elevation or depression. In observations to terrestrial objects this secant will be very nearly unity. The multiplication may be omitted if its effect is to change the mean reading by less than 0·1 second. As the micrometer readings must never exceed 15 divisions (about 20 seconds) the secant may be ignored if the angular elevation or depression does not exceed 6 degrees. The conversion of micrometer readings to seconds of arc may easily be performed by means of a table prepared for the purpose, or by slide rule.

The headings must be properly filled in, and the hour should also be recorded once on each page. The observer and recorder must sign the record.

At each station duplicate angle books are prepared on unbound sheets, which are sent in to the Director as soon as completed. They are prepared by copying the actual observations from the original, but the means are computed independently and are compared with the original.

56. Abstract.—The abstract Form 6 Trian (see fig. 16) must be kept up-to-date from hour to hour as the observations proceed. Prompt entries in this form are of great assistance when bad visibility makes it impossible to observe all angles in their regular order.

When rounds have to be broken (see para 49), and some angles deduced, measures of the sum of two angles should be entered separately in Form 6 Trian, as a new angle, the deduced values of the indirectly observed angle being entered in their proper places prefixed by the letter D. Suppose two angles ADB and BDC are being measured (see fig. 11), and that owing to the invisibility of B, some measures have been made of ADC, supplemented by later measures of BDC on the missing zeros. Then the rules for deducing and recording the missing values of the angles ADB are as follows:—

(1) Measures of BDC must never be combined with measures of ADC on a different zero.

(2) Measures of BDC made during a round in which ADB was included must not be used in computing deduced values of ADB.

(3) If on any one zero the number of measures of BDC made separately from measures of ADB is equal to the number of measures of ADC, the individual measures should

be combined in the order in which they were made, to give an equal number of deduced measures of ADB. These measures should be entered in 6 Trian under ADB just as if they were direct measures, but prefixed by the letter D.

(4) If the number of available* measures of BDC and ADC on any zero are unequal, the zero mean of these measures of BDC should be subtracted from the zero mean of ADC, and the difference should be entered in the appropriate column under ADB, prefixed by the letter D and a number in brackets indicating the number of measures of BDC or ADC, whichever is least. When computing the zero mean, this deduced value will be given weight corresponding to the number of measures.

The entries in Form 6 Trian will ordinarily be single measures, not means of FL & FR, unless systematic difference between faces makes it advisable to enter their means only. Such cases should be clearly indicated in the form.

All measures which the observer may consider fit for rejection should be entered in the abstract form where they should be lightly ruled out, and prefixed by the letter R. The reason for rejection should be recorded in the angle book.

The final mean values of all angles must be completed and checked before any station is left. If the observations at that station complete the three angles of any triangle, the sum of the three angles must be compared with a preliminary value of the spherical excess, and the triangular error noted, so that re-observations can be undertaken at once if required (see para 59). For the computation of the spherical excess a rough value of the area of the triangle can be obtained from the chart and used in conjunction with Table 1 Sur. At a central station the discrepancy between the sum of all the angles and 360° should also be noted.

Single measures will be recorded to the nearest tenth of a second. Zero means and the general means will be computed to the hundredth of a second.

The abstract form will be computed in duplicate and one copy will be sent to the Director as soon as it is prepared.

57. Rejections.—In spite of the greatest care on the part of the observer an observation will sometimes be recorded which is clearly bad. If the cause of the badness is obvious, such as the observer having kicked the stand of the theodlite, the fact should be noted in the angle book, and the readings which are affected should be rejected and repeated. It will sometimes happen that the observer calls out the wrong number of degrees or minutes, and

* See (1) & (2) on page 51.

Abstract of Horizontal Angles Observed at Rājkot *H. S.* 2 - Micro.

Angle between	Seconds of observation 0°	18°	36°	54°	72°	90°	108°	126°	144°	162°
Sultānpur H.S.	32·6″	31·6″	36·2″	33·8″	32·8″	31·8″	36·3″	33·8″	33·4″	33·9″
and	33·2	34·1	35·3	35·3	30·4	32·6	34·7	34·9	32·8	35·7
	31·1	30·8	36·1	34·9	31·9	30·2	35·0	32·9	34·0	34·4
Khānpur H.S.	30·7	33·2	34·0	34·2	33·3	32·8	33·9	35·4	31·9	36·2
	32·4	32·9	35·1	35·0	32·7	33·9	34·8	32·7	32·7	33·8
31° 16′	33·8	33·2	34·9	35·5	31·5	32·7	35·7	33·5	33·3	34·9
	·	·	·	·	·	·	·	·	·	·
No. of observations ...	6	6	6	6	6	6	6	6	6	6
Greatest Difference ...	3·1	3·3	2·2	1·7	2·9	3·7	2·4	2·7	2·1	2·4
Zero Mean ...	32·32	32·63	35·27	34·78	32·10	32·33	35·07	33·87	33·02	34·82
General Mean 33·62 ...										
Khānpur H.S.	03·9″	06·8″	04·2″	05·6″	07·4″	06·3″	02·9″	03·2″	07·6″	03·8″
and	04·1	05·0	05·4	07·2	05·8	04·7	05·4	05·8	04·8	04·3
	07·2	04·1	03·2	08·7	04·2	05·8	06·2	07·0	03·5	06·9
Adilabād H.S.	05·9	05·4	03·9	07·6	06·3	06·4	02·8	04·1	05·2	05·2
	04·8	03·6	05·2	05·4	03·4	07·5	05·9	03·5	04·5	03·4
47° 24′	03·1	06·2	03·4	06·6	07·0	05·8	03·4	06·2	06·2	04·8
	·	·	·	·	·	·	·	·	·	·
No. of observations ...	6	6	6	6	6	6	6	6	6	6
Greatest Difference ...	4·1	2·7	2·2	3·3	4·0	2·8	3·4	3·8	4·1	3·5
Zero Mean ...	04·83	05·18	04·22	06·85	05·68	06·08	04·43	04·97	05·30	04·73
General Mean 05·23 ...										
H.S.	·″	·″	·″	·″	·″	·″	·″	·″	·″	·″
and	·	·	·	·	·	·	·	·	·	·
	·	·	·	·	·	·	·	·	·	·
H.S.	·	·	·	·	·	·	·	·	·	·
	·	·	·	·	·	·	·	·	·	·
° ′	·	·	·	·	·	·	·	·	·	·
	·	·	·	·	·	·	·	·	·	·
No. of observations ...										
Greatest Difference ...	·	·	·	·	·	·	·	·	·	·
Zero Mean ...	·	·	·	·	·	·	·	·	·	·
General Mean ...										

Observed by A.B.C. *Examined by* D.E.F.

Abstracted by G.H.I. *Compared by* D.E.F. *and* G.H.I.

6 Trian.

Survey of India.

Fig. 16 (b)

Abstract of Horizontal Angles Observed at Taungnio *H. S.*

3 - Micro.

Angle between	00° L	00° R	132° L	132° R	264° L	264° R	36° L	36° R	168° L	168° R
	Seconds of observation									
Ayodaung H.S.	38.2″	34.9″	35.6″	39.2″	36.9″	40.1″	39.0″	35.9″	40.2″	37.9″
and	36.4	37.4	36.2	40.9	37.8	39.8	38.7	36.1	39.7	38.0
	37.5	35.4	36.4	41.6	38.4	38.8	37.5	36.1	42.1	37.5
Pegu H.S.	37.3	36.2	36.4	40.2	37.4	39.8	38.9	35.5	40.8	41.4
	38.4	39.9	35.9	42.6	37.2	37.5	37.6	37.6	37.9	36.5
87° 10′	38.4	37.6	38.1	42.3	35.7	38.4	38.1	38.1	40.8	37.4
	.	.	.	.	.	.	.	.	.	.
No. of observations ...	6	6	6	6	6	6	6	6	6	6
Greatest Difference ...	2.0	5.0	2.5	3.4	2.7	2.6	1.5	2.6	4.2	4.9
Zero Mean ...	37.70	36.90	36.43	41.13	37.23	39.07	38.30	36.18	40.25	38.12
General Mean 38.13 ...										
Pegu H.S.	33.7″	35.3″	35.4″	31.8″	33.9″	33.5″	32.5″	34.2″	33.7″	33.2″
and	34.3	34.1	34.1	31.9	32.1	34.6	32.3	32.1	35.2	31.4
	32.5	35.3	34.9	33.2	33.0	33.5	32.8	31.4	34.9	32.6
Mako H.S.	34.5	35.5	34.3	32.2	33.2	32.6	31.0	35.1	35.8	33.5
	29.2	34.0	36.7	34.9	32.5	33.0	33.8	33.5	36.7	30.9
22° 40′	35.0	36.2	33.4	31.9	31.8	34.2	30.4	32.8	33.2	29.7
	.	.	.	.	.	.	.	.	.	.
No. of observations ...	6	6	6	6	6	6	6	6	6	6
Greatest Difference ...	5.8	2.2	3.3	3.1	2.1	2.0	3.4	3.7	3.5	3.8
Zero Mean ...	33.20	35.07	34.80	32.65	32.75	33.57	32.13	33.18	34.92	31.89
General Mean 33.42 ...										
________ H.S.	″ .	″ .	″ .	″ .	″ .	″ .	″ .	″ .	″ .	″ .
and	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
________ H.S.	.	.	.	.	.	.	.	.	.	.
° ′	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
No. of observations ...										
Greatest Difference ...	.	.	.	.	.	.	.	.	.	.
Zero Mean ...	.	.	.	.	.	.	.	.	.	.
General Mean ...										

Observed by A. B. C. *Examined by* G. H. I.

Abstracted by D. E. F. *Compared by* D. E. F. *and* G. H. I.

that the error is not noticed until the theodolite has been turned on to the next station. In such a case, provided the measure of the seconds agrees with others within the usual limits, the presumed correct value of the degrees or minutes may be entered with a query over the recorded value, and means may be taken as if this value had actually been read. But if the seconds reading is at all wide from the mean, the measure should be rejected and repeated.

Apart from such clear cases as the above, it will occasionally happen that a value is recorded which is very wide from the mean, but which otherwise appears to have been perfectly well observed. The ideal treatment of such cases is firstly that no observation should be rejected solely because it does not agree with others, and secondly that when an exceptionally wide observation does occur it should be swamped by so many repetitions that its rejection or retention will have a negligible effect on the mean value.

The rule that wide observations are to be retained and swamped with more normal ones, must not of course be pushed to absurd lengths: for instance, if the six measures of an angle made on one zero of a certain theodolite are almost invariably found to lie within two seconds of the mean, and if on a certain occasion the seconds of the measures are recorded as 25″·2, 27″·2, 23″·8, 39″·8, 25″·7, 24″·9, in this case it is not practicable to swamp the value 39″·8, and this value should be rejected, two repetitions should be made, and in every case the occurrence should be brought to the notice of the observer's immediate superior. Such cases as the above should be extremely rare (certainly less than 1°/₀ of the total readings); if they are more frequent something is wrong, either with the instrument or with the atmospheric conditions or with the observer.

There is no case for the rejection of an observation unless it is quite isolated in its badness from those which lie nearest to it. If a series of measures show differences from the mean of 1″·2, 0″·5, 3″·5, 6″·1, 0″·3, 2″·8, 0″·6, 4″·9, 0″·0, there is no case for the rejection of 6″·1, because it is not isolated, although if a 12-inch theodolite produced such irregular results, it might well be considered that something was wrong, and the whole group might be cancelled and repeated when the cause of the error had been found or when conditions were more favourable. Another case in which rejections are sometimes improperly made is when a number of observations happen to fall exceptionally close together, while one falls wide of them, but not wider than may happen with the type of instrument in use. Thus if a series of measures with a 12-inch theodolite results in 23″·8, 23″·7, 25″·8, 23″·1, 24″·0 and 23″·0, there is no case for the rejection of 25″·8, for readings on this instrument will generally range through as much as 3″, and in this case the close agreement of the other five must be considered to be due to chance.

58. Repetitions.—After obtaining some experience of his instrument the observer will discover a standard within which his intersections, measures, and zero means will generally agree. When individual values fall wide of this standard, repetitions must be made.

In the case of intersections the usual rules for repetitions are given in para 48. A common fault is to work to some rather close standard e.g., that the three values must agree within two seconds, and when this is not fulfilled to make repetitions until three accordant values are obtained, and then to retain these and reject the rest. This is a bad practice: rejections should only be made in accordance with para 57, and it will very seldom be proper to reject an intersection with which the triangulator was satisfied at the moment of its making. Any progressive movement of the mark, indicated by (say) a steady increase in a series of intersections, should be carefully watched. Under such conditions, it will probably be best to cease observations until a more favourable hour.

In the case of the several measures on one zero, it is usual for 90°/₀ to be within 2″ or 3″ of the mean in primary work, provided there is no systematic difference between FL and FR, in which case the measures must be meaned in pairs, and the accordance of the means considered*. As a general rule the number of measures should be increased in proportion to any increase in this discrepancy: e.g., if more than one measure is found to be 4″ or 5″ from the mean a total of 12 measures should be made on that zero instead of 6. But if such extensive repetitions are often found necessary, and if the cause cannot be found and remedied, the triangulator should work to some lower standard. Occasional discrepancies of 3″ or even 4″ at this stage are not incompatible with the attainment of the highest accuracy ($m = 0''\cdot 3$) in the general mean.

In a similar manner 90 per cent of the different zero means should generally lie within 2½″ of the general mean. If they fail to do so, it will not be practicable to make repetitions except in the case of outstandingly bad zeros, which may be repeated twice and then either rejected (after reference to the Director) or else meaned with the repetitions. Thus suppose the zero means are 48″·2, 50″·7, 49″·3, 50″·2, 49″·1, 48″·8, 54″·0, 51″·8, 50″·0, 51″·9, 50″·1, 50″·9. Mean 50″·4. The outstanding value of 54″·0 may be repeated, with results of perhaps 51″·0 and 52″·1, and the mean of all three, viz., 52″·4, may be accepted in its place.

* Provided the instrument is in fair adjustment, this systematic difference can only occur in astronomical work.

It should seldom be necessary to make such repetitions; at most, they should occur in not more than one or two zeros per station. Zeros should only be repeated in this way if they are exceptionally wide of the mean; it is not right always to repeat those zeros which happen to be furthest from the mean. It is obviously improper to attempt to reduce a (say) positive triangular error by making uncalled-for repetitions of all or any of the zero means which are greater than the general mean. If most angles contain more than one zero mean which is more than two seconds from the general mean, it will be necessary to observe on more zeros, if the highest accuracy is to be obtained (i.e., $m = 0''\cdot 3$), unless the theodolite is known to have a regular systematic graduation error of this amount.

59. Large Triangular Error.—When an unduly large triangular error is obtained the triangulator is in a very difficult position. To re-observe all three angles of the triangle is not only a costly operation in itself, but it will completely disorganise all the arrangements which he has made with his signal squads.

Before leaving for the field, instructions must be obtained from the Director as regards (1) what triangular error is definitely allowable, and (2) what triangular error calls for repetition. The answer to these questions will depend somewhat on the type of country, and the importance of the series, but as a general rule a triangular error of less than one second can be accepted without doubt, while one of two seconds calls for repetition, unless the whole triangle can be omitted without great loss of strength. But repetition is not to be carried out unless the observer has discovered, or at least strongly suspected, and remedied, the source of the error. The statement that an error of one second is readily allowable, must not be misunderstood to mean that the average should be as high as this.

Between the allowable and disallowable values (e.g., between one and two seconds) lies a range of triangular errors, regarding which the triangulator will have to use his discretion. If his earlier triangles have all closed very well (say less than $0''\cdot 5$) he will be more ready to re-observe than if the previous triangular errors have been rather near the limit. For instance if triangular errors are $0''\cdot 8$, $1''\cdot 2$, $0''\cdot 3$, $0''\cdot 9$, $1''\cdot 1$ and $1''\cdot 5$, there is no reason to re-observe the $1''\cdot 5$ triangle. The series is not of the best quality and it will not be improved by the re-observation of the triangle which happens to have the largest closing error. Re-observation will be more readily undertaken in country where communications are easy, than in difficult country, and if the resulting delay will not prevent the completion of the season's programme.

On discovering a large triangular error, the observer must discover the cause. Unless the cause is remedied, re-observation is useless. With a 12-inch or Wild theodolite in good order, with rays which are generally well clear of the ground, and with favourable atmospheric conditions, triangular errors of two seconds cannot occur unless there has been some departure from the rules laid down in this handbook. The observer should first consider his abstract of angles and see whether the various measures and zeros have been agreeing within reasonable limits. If they have not, either the theodolite is in bad order, or else there is irregular lateral refraction, or in trestle stations the trestle may be responsible. Such irregularity should, of course, have been noticed as soon as it occurred, and remedied if possible. Possible faults in the theodolite are enumerated in para 60. Lateral refraction of serious extent is only likely to occur in grazing rays. If the graze is enforced by the flatness of the country, it can only be remedied by shorter rays or higher trestles. If the trouble occurs in one or two rays only, it may perhaps be remedied by one or two extra stations: but if the whole series is involved, it will probably be best to recognise that the highest accuracy is not obtainable, to be content with comparatively low class angular measures, and if necessary to arrange for an extra base and Laplace station. The Director's orders should be obtained as early as possible. In hilly country grazes will not occur if the reconnaissance has been properly carried out, except that occasionally the bad siting of an old series will involve a difficult start. The remedy is generally to put an extra station near the point where the graze occurs (see Appendix I), as should already have been done (see para 10). It may happen that the resulting triangle contains an extremely short side, less than the 3 miles allowed by para 8. This is a weakness, but is better than a serious graze. If the most careful centering cannot avoid serious angular error, this small triangle will probably have a large triangular error. In this case the error must be distributed between the two large angles, and the small angle must be left alone. The azimuth of forward rays must not be deduced from a short side of less than one mile, but from the long sides only. In very short sides special marks will be read such as a nail in front of a piece of white paper, and such marks should be raised so as to be close below the theodolite.

Irregularity of observations due to the unsteadiness of a trestle station can only be remedied by building a better or a lower trestle. It should be noticed before the station is vacated.

If the angles in the abstract are accordant among themselves, a large triangular error is very unlikely. It can only be due to centering error of either signals or theodolites, stiffness of the axis of the theodolite resulting in all measures being too small, or to a grazing ray past a hill which is always hotter or colder than the surrounding air. Centering error is a very unlikely cause, unless masts and trestles have been employed. The centering of lamps or helios is almost a foolproof operation, and on arrival at each station, the observer will have seen the lamp and helio in position on the mark. It may be remembered that one foot subtends one second at a distance of 40 miles. Stiffness in the axis is extremely unlikely to give a serious error in the mean unless it also causes noticeable raggedness among the individual measures: its presence can be verified as explained in para 46. Grazing rays are discussed above.

Any re-observation at a station which is revisited on account of bad triangular error, must consist of a complete programme. It will not suffice to re-observe selected zeros only. Cases of re-observation should be reported at once to the Director, but it will be seldom that he will be able to give orders or advice in time for them to be of any use.

60. Theodolite faults.—Possible faults in the theodolite which may be responsible for discrepancies between different readings (see para 59) are :—

(*a*) Stiffness in the axis, or slackness in the joints, which may be verified as explained in para. 46.

(*b*) Graduation error. This may be verified by analysing the angles to see if all angles measured on a certain part of the circle tend to be greater or less than the general mean. It must be noted that graduation error cannot develop suddenly, except as the result of an accident. If it has been small one season, it is unlikely to be very bad in the following year.

(*c*) Looseness of the object-glass, diaphragm or micrometer microscopes, or dirt in the pivot bearings; *i. e.*, inconstancy in the relation of the line of collimation to the zeros of the micrometers.

(*d*) Neglect to level the theodolite, to correct the runs etc., of the micrometers, or to eliminate parallax either in the eye-piece or the micrometers. Non-verticality of the vertical wire.

(*e*) Neglect to make all intersections against the springs of the slow motion screws or micrometers. Or neglect to swing continuously through each round of angles.

(*f*) Use of a grossly inaccurate value for one division of the eye-piece micrometer.

(*g*) The practice of moving the reflectors or prisms which illuminate the circle, while an angle is being measured. Or the careless illumination of the circle of a 12-inch theodolite at night.

61. Procedure at a station.—The following is a summary of the procedure of the observer at each station:—

(*a*) Immediately on arrival he will see that the lamp and helio are properly centered. Orders must always be given that they are not to be moved until inspected.

(*b*) He will then see that the station is properly constructed, that the necessary clearing has been done, and that there is no obvious obstruction in the line of sight.

(*c*) Surrounding stations should be called up.

(*d*) If the hours of minimum refraction are not yet over, the observer may consider the advisability of observing vertical angles before the observatory tent is pitched. But it must be pitched before horizontal angles can be begun.

(*e*) The observer should himself supervise the pitching of the observatory tent.

(*f*) The theodolite adjustments should be tested and the runs recorded.

(*g*) Observations should be made. They should extend over at least two days.

(*h*) All means must be taken out, checked and examined, and any possible triangular errors computed before *jawābs* are given.

(*i*) *Jawābs* are given to the surrounding stations (see para 67).

(*j*) The observer will see that sufficient stones have been collected to form a cairn (see para 25).

(*k*) He will watch the station signal squad replace their lamp and helio, and give them instructions regarding their future movements.

62. Description of station.—A full and accurate description of the station must be entered in the angle book. It must describe the structure of the station, with special reference to the relative position of the mark-stones, so that if one mark-stone is found in later years when the station is half ruined, there may be no doubt whether it is the upper, middle or lower mark. The auxiliary marks must also be carefully described, and their positions illustrated in a dimensioned plan.

It must describe the position of the station so clearly that it may be possible to identify the site in 100 years time, when most of the surrounding villages will have moved, and their names will have been forgotten. This description presents little difficulty if the station is on the top of the locally highest hill.

It must describe the routes by which the station is most easily reached from different directions: it must give an idea of their difficulty, and for what kinds of transport they are practicable: it must mention the nearest source of transport and supplies.

Specifically, it must mention the village, tahsil, district and province in which the station lies, and it must give the names, directions and distances of 4 or 5 well known neighbouring villages, temples, hills etc. Names must be clearly written, and care must be taken in the transliteration of native words. It will always be desirable to have the names written in the vernacular by a local official.

The name of the station will be that of the hill on which it stands, if a distinctive name exists. Otherwise it will be the name of the nearest village.

63. Satellite stations.—If for any reason, such as the use of towers and trestles, the signal or theodolite are not exactly plumbed over the station mark, the horizontal distances and azimuths from the mark must be measured, and must be recorded clearly in the angle book, with sketches showing their relative positions. It must also be stated from what stations the satellite signal has been observed, and to what stations observations have been made from the satellite station. The reductions to centre must be carried out in the field by the observer himself. They must be carried out in such good time that triangular errors can be checked before leaving the third station of each triangle.

64. Signals.—The Argand lamp (see fig. 17) is the one now in use. It is of heavy and primitive construction, but it is reliable and foolproof, and can be seen at a distance of 50 miles through ordinarily clear air. Detachable boards are screwed to the top of the box, which form a platform for the helio used by day. The lamp is centered by placing the front foot in the station mark. The lamp is then aligned by means of two small sights on the upper side of the centre helio board, and the foresight is plumbed over the centering foot by means of a plummet and the two back footscrews. The alignment must of course be checked after plumbing. The lamps and mirror are fixed in the box, and unless the lamp has been damaged, the alignment is now correct.

The wicks must be kept carefully trimmed, and with long rays they must be turned up as far as is possible without smoking. The glass front must be kept shut, and the special flattened glass chimney must be turned so that its flat sides are towards the front and back. The reflector must be touched as little as possible, but when dirty it should be very gently cleaned with soft clean paper of which a supply should be carried. The oil consumption is about 6 nights to the gallon of kerosene oil, of which only the best quality should be used.

Helios are fully described in the Handbook of Topography, Chapter III, and the primary triangulator is presumably well acquainted with their use. In primary triangulation it is usual for a signal squad to carry a 9 or 12-inch and a 6-inch helio, the former for ordinary use and the latter for use when double reflection is required. The 9-inch helio is stood in three brass grooves on the centre board of the lamp. A frame carrying the cross-wires is screwed on to the front of the lamp box. The lamp must first be aligned by its own sights, and plumbed. It is then necessary that the station and cross-wires should be seen in alignment through the helio. This is arranged by the help of the foot-screws of the helio, by which its centre can be raised or lowered, or tilted sideways. For double reflection the small helio is stood on one of the side boards. Its shadow is directed on to the centre of the large helio, whose shadow is directed on to the cross-wires.

It is a common fault among inexperienced *khalāsis*, that they will take great care to keep the shadow properly on the cross-wires, but that they will not get the initial alignment correct. A bad initial alignment will result in the helio never being seen, whereas carelessness with the shadow will at least give an intermittent light. The mirrors must be kept clean.

The Hunter portable mast signal has been used in recent series passing through flat country (see para 22). It can be used as an opaque signal, or a lamp can be hoisted to the top, or a helio may be used as described in Appendix I. Owing to the impossibility of aligning a lamp correctly, a high candle power pressure-fed lamp should be used instead of an oil lamp and reflector.

65. Testing lamps.—The lamps contain numerous small detachable parts, and they must be assembled by their lampmen and examined before leaving headquarters. No adjustment is provided for regulating the relative positions of lamp, reflector, sights and centering foot, but their adjustment should be tested at the beginning of the season. A theodolite should be set up by day about 50 yards in front of the lamp, whose sights should be aligned

The wicks must be kept carefully trimmed, and with long rays they must be turned up as far as is possible without smoking. The glass front must be kept shut, and the special flattened glass chimney must be turned so that its flat sides are towards the front and back. The reflector must be touched as little as possible, but when dirty it should be very gently cleaned with soft clean paper of which a supply should be carried. The oil consumption is about 6 nights to the gallon of kerosene oil, of which only the best quality should be used.

Helios are fully described in the Handbook of Topography, Chapter III, and the primary triangulator is presumably well acquainted with their use. In primary triangulation it is usual for a signal squad to carry a 9 or 12-inch and a 6-inch helio, the former for ordinary use and the latter for use when double reflection is required. The 9-inch helio is stood in three brass grooves on the centre board of the lamp. A frame carrying the cross-wires is screwed on to the front of the lamp box. The lamp must first be aligned by its own sights, and plumbed. It is then necessary that the station and cross-wires should be seen in alignment through the helio. This is arranged by the help of the foot-screws of the helio, by which its centre can be raised or lowered, or tilted sideways. For double reflection the small helio is stood on one of the side boards. Its shadow is directed on to the centre of the large helio, whose shadow is directed on to the cross-wires.

It is a common fault among inexperienced *khalāsis*, that they will take great care to keep the shadow properly on the cross-wires, but that they will not get the initial alignment correct. A bad initial alignment will result in the helio never being seen, whereas carelessness with the shadow will at least give an intermittent light. The mirrors must be kept clean.

The Hunter portable mast signal has been used in recent series passing through flat country (see para. 22). It ...

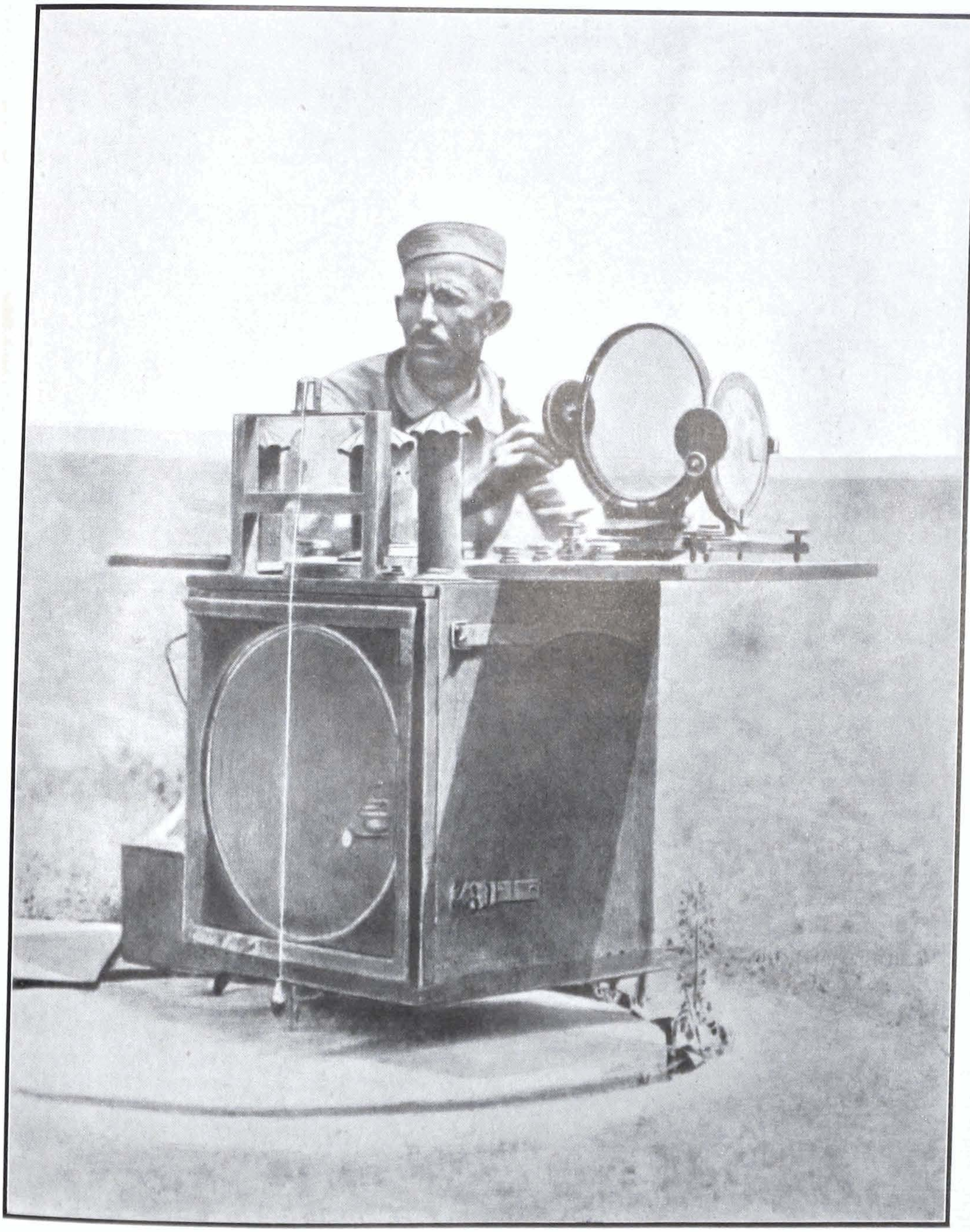
Page 60, line 33.

For Appendix I *read* Appendix II.

No. 5 dated 8-6-31.

... parts, and they must be assembled by their lampmen and examined before leaving headquarters. No adjustment is provided for regulating the relative positions of lamp, reflector, sights and centering foot, but their adjustment should be tested at the beginning of the season. A theodolite should be set up by day about 50 yards in front of the lamp, whose sights should be aligned

Fig. 17.



Argand Lamp with Helio.

on a mark about one foot above the object-glass of the theodolite. The foresight should be plumbed over the foot. Then on examination through the theodolite the centering foot, the centre of the wick, the foresight, and the helio sight, should all appear in a vertical line, and the right and left hand edges of the reflector should appear equidistant from this line. Any error in the position of the helio sight or of the reflector will result in a centering error. Error in the position of the box foresight or lamp will result in diminished illumination. The lamps should then be lit, and the whole mirror should appear brightly and evenly illuminated. Uneven illumination will of course amount to a centering error and will also mean that the best light is not being obtained. If a better light is apparent from any other position above, below, or to the side of the theodolite, the mirror is not properly adjusted. This test cannot be carried out until late in the evening.

It should be exceptional that any serious error is found, as the relative position of the parts are all well fixed, and an error of one-tenth of an inch will have no significance. Nevertheless, the triangulator must not accept their accuracy without proof.

66. Strength of detachment.—The normal strength of an observing detachment (excluding reconnaissance detachment) is one observer, one recorder, eight signal squads of four men each, and fifteen men with the observer if a Wild theodolite is used, or eight more for a 12-inch theodolite. Some increase will be required if trestles and masts are extensively employed. In Burma and other countries with strange languages an interpreter will be engaged.

If two observers are available, it may sometimes be more economical for them to work together, each on one flank of a series, than it will be for them to begin separately at opposite ends of the series. The advantages of this are a considerable saving in lamp squads, and the opportunity given for an inexperienced triangulator to get advice and assistance from the other observer. The disadvantages are the risk that bad work or delay by one observer will upset the work of the other, that movement of signal squads are rather more complicated, and that responsibility is divided. There is also the difficulty of showing a signal from a station on which a theodolite is working, and of showing a signal from one station to two others simultaneously. This method of work has not been adopted by the Survey of India, but it should be considered every year whether the circumstances and personnel available do not make it desirable.

67. Movement of signal squads.—One of the most difficult and important cares of the triangulator is to ensure that on his arrival at a station he will find all the surrounding stations occupied by signal squads, ready to direct their lights on to his station. This can only be ensured by the most carefully thought out system of movements. Before commencing work the triangulator must prepare a programme showing the movement of himself and all his squads throughout the season, including the movements of any messengers which he may have to send with instructions. He must consider what transport will be required, and how the squads are to find the stations. This programme cannot accurately show the dates of each movement, but it should show the movements which are to take place on the completion of work at each station. It may sometimes be necessary to make changes in the programme, if the reconnaissance is still in progress, but so far as the positions of stations are known it should be so carefully thought out that changes will very seldom occur.

Lampmen cannot be given complicated instructions. Generally the most complicated instructions that can be given are that they are to go to such and such a station; that from there they will be required to direct their lights to certain stations in the order named, and that on receiving a final *jawāb* (see below), they will proceed to such and such a station or place. Especially intelligent men with a little knowledge of writing may receive longer instructions, but to attempt it with most men is to risk confusion.

By means of his helio the observer can communicate with his lampmen. There are three recognised signals :—

(*a*) A flashing of the helio once or twice a second means "Your light is not showing; show it". The flashing should be continued for a sufficient length of time for the distant helio man to align his helio on it. This signal should not be given unnecessarily, as it causes confusion in the minds of the lampmen. It should not be given because the helio becomes temporarily faint or invisible; that may be due to cloud, or to a temporary slackness in directing the shadow; if the latter, it should be corrected by warning next time that signal squad is met, or by a message. It must not be corrected by perpetual flashing.

Ideally the station helio should only be used to call up on arrival at the station, and to give *jawāb* on completion.

(*b*) On completion of the work at a station, the helio is directed steadily on to the distant station. The lampman, recognising the signal, puts out his helio, generally by covering it with his *pagri*. The station helio is then similarly dimmed, and the distant lampman uncovers his helio. The station helio is then uncovered and the distant helio again covered. The station helio is covered again and the distant helio uncovered. Once more the station helio is uncovered and the distant helio covered. The distant helio then gives a few flashes to show that he has understood the signal. This is known as the "*three jawāb*". Its meaning is "I have completed work at this station. Stay where you are, and be ready to show a light to my next station".

This signal is given to all surrounding stations in turn, on completion of work at each station.

(*c*) As above the station helio is directed on to the distant station, and the process of uncovering and covering is repeated nine times. Its meaning is "I have finished work at this station. You are to leave your present station and go to the place previously arranged".

The giving of *jawābs* in cloudy or hazy weather is always difficult. They may be given by lamp, but helio is generally preferable. It must be remembered that the lampman has no telescope. On the other hand the observer's helio has the advantage of being perfectly aligned, by the observer himself if necessary. If the lampmen are not being perpetually worried by flashing for a better light, they will be ready to receive *jawābs* whenever they see the station light, and will probably understand it, however imperfectly it may be seen. If clouds interrupt a *jawāb*, it should not be repeated at once, lest a repeated 3 *jawāb* be mistaken for a 9 *jawāb*.

It has sometimes been customary to give a single *jawāb* at the end of each day or night's work. This is a convenience to the lampmen, enabling them to rest and to save oil. But there is risk of confusion, and it should not be adopted.

Although signal squads will be told from what stations they are to expect a light, it is their duty to keep a good look-out in all directions while the observer is marching, in case the programme is changed.

Signal squads must be careful not to light their fire or pitch their tents in positions from which the observer can see them, or he may intersect them instead of the lamp. Nor must they allow smoke to obscure the line.

In some cases it would be very desirable for each signal squad to contain one man of better education, capable of reading and sending messages by Morse code in Roman Urdu. The squads could then be given orders without the need of sending messengers, and changes of programme could be arranged without fear of confusion. In high altitudes considerable hardship is involved in remaining on the station for weeks at a time, during half of which the observer is marching. If signals could be sent, permission could be given to leave the station at such times.

68. Reasons for precautions.—The following is a summary of the reasons for the different precautions and adjustments which the observer has to carry out :—

(*a*) Change of face. To eliminate errors of collimation, or dislevelment of the transit axis.

(*b*) Change of zero. To eliminate errors of graduation.

(*c*) Reading at least two microscopes. To eliminate errors due to the vertical axis not passing through the centre of the graduated circle.

(*d*) Setting zero carefully to odd minutes (see para 48). To eliminate errors of micrometer run.

(*e*) Continuous direction of movement when swinging the telescope. To eliminate errors due to stiffness in the axis combined with elasticity or looseness in the theodolite or stand.

(*f*) Face left, swing left. For convenience, and to ensure that there are an equal number of swing left and swing right measures.

(*g*) Day and night observations. To reveal systematic errors due to horizontal refraction.

(*h*) Use of observatory tent. To lessen temperature changes in the instrument and stand, and so to lessen the movement of the instrument during the course of the measure of an angle.

(*i*) Intersection with moving wire (in 12-inch theodolite). To increase the number of intersections with very little effort. This is especially desirable if the signals are unsteady.

(*j*) Light clamping of the vertical circle while reading horizontal angles. To avoid mechanical strain in the horizontal axis with consequent distortion and tendency to ride out of the Y's. Clamping is desirable in order to bring the mark close to the horizontal wire.

(*k*) Last movements of slow motion and micrometer screws to be against the springs. To avoid back-lash. Also see para 51(*g*).

(*l*) Careful centering. It is necessary for short rays, and is an example to *khalāsis* who have to centre their lamps and helios.

(*m*) Levelling. To ensure that the projection of the measured angle is on to a horizontal plane. If an angle with an elevation a is measured with the theodolite dislevelled by a tilt of θ at right angles to the ray, the horizontal pointing will be in error by $\theta \tan a$. The error does not cancel with change of face or zero.

(*n*) Collimation, and levelling of the transit axis. To obtain agreement between FL and FR measures, for mutual check, and easy meaning.

(*o*) Micrometer run. Errors of run cause identical errors in all measures made on the same zero: there is no cancellation. If the zero settings are all a whole number of degrees, there is no cancellation of micrometer errors on different zeros, but if the settings are made in accordance with para 48, a considerable cancellation occurs.

(*p*) Agreement between zeros of micrometer drums, combs, and rough reading miscroscopes. For convenience, and to avoid misreadings of whole minutes.

changes of programme could be arranged without fear of confusion. In high altitudes considerable hardship is involved in remaining on the station for weeks at a time, during half of which the observer is marching. If signals could be sent, permission could be given to leave the station at such times.

68. Reasons for precautions.—The following is a summary of the reasons for the different precautions and adjustments which the observer has to carry out:—

(*a*) Change of face. To eliminate errors of collimation, or dislevelment of the transit axis.

(*b*) Change of zero. To eliminate errors of graduation.

(*c*) Reading at least two microscopes. To eliminate errors due to the vertical axis not passing through the centre of the graduated circle.

(*d*) Setting zero carefully to odd minutes (see para 48). To eliminate errors of micrometer run.

(*e*) Continuous direction of movement when swinging the telescope. To eliminate errors due to stiffness in the axis combined with elasticity or looseness in the theodolite or stand.

(*f*) Face left, swing left. For convenience, and to ensure that there are an equal number of swing left and swing right measures.

(*g*) Day and night observations. To reveal systematic errors due to horizontal refraction.

(*h*) Use of observatory tent. To lessen temperature changes in the instrument and stand, and so to lessen the movement of the instrument during the course of the measure of an angle.

(*i*) Intersection with moving wire (in 12-inch theodolite). To increase the number of intersections with very little effort. This is especially desirable if the signals are unsteady.

(*j*) Light clamping of the vertical circle while reading horizontal angles. To avoid mechanical strain in the horizontal axis with consequent distortion and tendency to ride out of the Y's. Clamping is desirable in order to bring the mark close to the horizontal wire.

(*k*) Last movements of slow motion and micrometer screws to be against the springs. To avoid back-lash. Also see para 51(*a*).

Page 64, last line.

For para 51 (*g*) *read* para 51 (*p*).

No. 6 dated 8-6-31.

(*l*) Careful centering. It is necessary for short rays, and is an example to *khalāsis* who have to centre their lamps and helios.

(*m*) Levelling. To ensure that the projection of the measured angle is on to a horizontal plane. If an angle with an elevation α is measured with the theodolite dislevelled by a tilt of θ at right angles to the ray, the horizontal pointing will be in error by $\theta \tan \alpha$. The error does not cancel with change of face or zero.

(*n*) Collimation, and levelling of the transit axis. To obtain agreement between FL and FR measures, for mutual check, and easy meaning.

(*o*) Micrometer run. Errors of run cause identical errors in all measures made on the same zero: there is no cancellation. If the zero settings are all a whole number of degrees, there is no cancellation of micrometer errors on different zeros, but if the settings are made in accordance with para 48, a considerable cancellation occurs.

(*p*) Agreement between zeros of micrometer drums, combs, and rough reading miscroscopes. For convenience, and to avoid misreadings of whole minutes.

SECTION IV

COMPUTATIONS

69. Summary.—As the result of the operations described in the previous section, the observer produces an abstract of all his angular measures, from which he derives a general mean value of each observed angle (see para 56). The object of the computations is to deduce the latitude and longitude of each station from these data. The heights of the stations are also deduced from the observed vertical angles.

The computations involve the following processes:—

(*a*) Ideally the observed values of the angles require certain small corrections, which in practice generally have to be ignored.

(*b*) The computation of the weights of the angles, and the figural adjustment.

(*c*) The computation of the lengths of all the sides of the triangulation.

(*d*) The computation of the latitudes and longitudes.

(*e*) The height computations.

(*f*) Adjustment of the series.

(*g*) Compilation of the final results.

Paras 70 to 77 give a theoretical outline of processes (*a*) to (*e*). Para 78 describes the practical routine for carrying out these computations. The adjustment of the series is described in para 85.

70. The reference spheroid.—The position of a point on the surface of a spheroid is defined by two angles known as its latitude and longitude. Figure 18 represents a spheroid, whose centre is at O and whose minor axis is RS. P is a point whose position is to be defined. PQ is the normal at P, that is to say a line drawn at right angles to the tangent plane at P. It necessarily intersects the axis RS, but does not generally pass through the centre O. Then the latitude of P is $90° - RQP$. The longitude is the angle AOB, where RBS is a plane passing through the polar axis and P, and RAS is a similar plane passing through any arbitrarily chosen point G (Greenwich).

If the earth was a perfect spheroid, the position of any point on its surface could be defined in this way. The actual surface of the earth is clearly not a perfect spheroid: it is covered with hills. The difficulty caused by these irregularities is easily got over by defining the co-ordinates of a station to be those of a point at sea-level vertically below it, for the surface of mean sea-level is a much closer approximation to a spheroid. The sea-level surface is only apparent in areas covered by sea, but it may be supposed to be continued inland by hypothetical canals, and this whole surface is called the Geoid. In practice its depth below the earth's surface at any point is determined by spirit levelling, which gives the heights of bench-marks above this hypothetical sea-level surface. The triangulator's particular interest in the geoid lies in the fact that when a theodolite is levelled, its vertical axis is perpendicular to the geoid, not to the spheroid, so that an inclination between the two surfaces is equivalent to dislevelment of his instrument. Although the geoid closely approximates to a spheroid it also has irregularities caused by the attraction of hills and irregularities in the density of the earth, so that although it is possible to define the position of a point by means of its geoidal (commonly called astronomical*) latitude and longitude, these co-ordinates are not a suitable basis for computations: for the astronomical meridians and parallels do not follow any exactly regular system: two parallels which are separated by 100,000 feet in one longitude may be separated by 99,000 feet in another.

For purposes of computation it is consequently necessary to define the position of a station as the latitude and longitude of a point vertically beneath it on the surface of some more or less arbitrarily defined spheroid of reference, a perfect spheroid whose meridians and parallels are subject to mathematical treatment. This spheroid is defined by the length of its two axes, by the north-and-south and east-and-west components of its inclination to the geoid at some point known as the origin, and by the vertical separation between it and the geoid at this or any other one point. Its minor axis is taken to be parallel to the axis of the earth's rotation. The orientation and axes of the spheroid of reference are naturally so chosen that the spheroidal surface is always fairly close to the geoid, but it is impossible for the two surfaces to coincide everywhere, and they are generally inclined to each other by some small angle called the "deflection" or "deviation of the vertical" which is seldom less than several seconds and which may be as much as one minute. (See fig. 19).

In the same way, the height of a point might be expressed as so many feet above the spheroid of reference, but since ordinary spirit levelling gives geoidal heights, to which ordinary triangulation also gives a close approximation, and since geoidal heights are of more

* The astronomical latitude of a point is the angle between the polar axis and that tangent to the geoid which meets the polar axis.

practical* importance than spheroidal, it is customary to describe the height of point as so many feet above the geoid.

The Indian spheroid of reference is Everest's spheroid. The lengths of its axes are 20,922,931·80 and 20,853,374·58 feet†. The deviation of the vertical at Kaliānpur origin is 0″·3 south and 2″·9 west (i.e. the inward geoidal normal lies S. and W. of the spheroidal), and in height the geoid and spheroid coincide at Kaliānpur‡. This spheroid was selected a hundred years ago, and it does not fit the geoid very well. In Baluchistān and North Burma, the two surfaces are separated by about 150 feet, and (apart from local irregularities) are inclined to each other at an angle of 10 to 15 seconds. This results in a certain amount of inaccuracy in the reduction of the triangulation, but a change of spheroid, involving a change in all published data, would cause a great amount of unnecessary expense and inconvenience, and Everest's spheroid is likely to remain in use indefinitely.

71. Preliminary corrections.—There are three § preliminary corrections which should ideally be applied to all angles, viz. :-

(*a*) For deflection.

(*b*) For height of station and point observed.

(*c*) To geodesic.

The first of these is generally by far the most important, but the data required for its application are generally unknown. Consequently it is customary to ignore the others, and the practical computer is not concerned with the contents of this paragraph. Nevertheless it is desirable that it should be understood what approximations are being made, and what is being ignored.

(*a*) *Deflection.*—As is well known, a theodolite set up at a station S does not measure the angle subtended by two stations A and B (see fig. 20) on which it is directed, but the horizontal projection of the angle, that is to say the angle A′ S B′, where AA′ and BB′ are vertical, or more precisely they lie in the plane of the vertical axis of the theodolite, which (if the instrument is level) lies in the normal to the geoid at the station of observation. For computational purposes the angle required is the projection on to the spheroid, viz. A″S B″, and it is clear from

* The geoidal height of mean sea-level is always zero. The spheroidal height of a point at mean sea-level may be + or − 50 feet, which would be very unsatisfactory for the contouring of maps.

† i.e. Indian feet, or tenths of Standard Bar A, which was the fundamental Standard at the time when the Indian bases were measured, and in terms of which all sides of the triangulation are expressed. See Profl. Paper No. 16 pages 1 and 2. by J. de Graaff Hunter.

‡ See Geodetic Report Vol. III, Pages 134 and 135.

§ See Departmental Paper No. 12, *Geodesy*, by J. de Graaff Hunter.

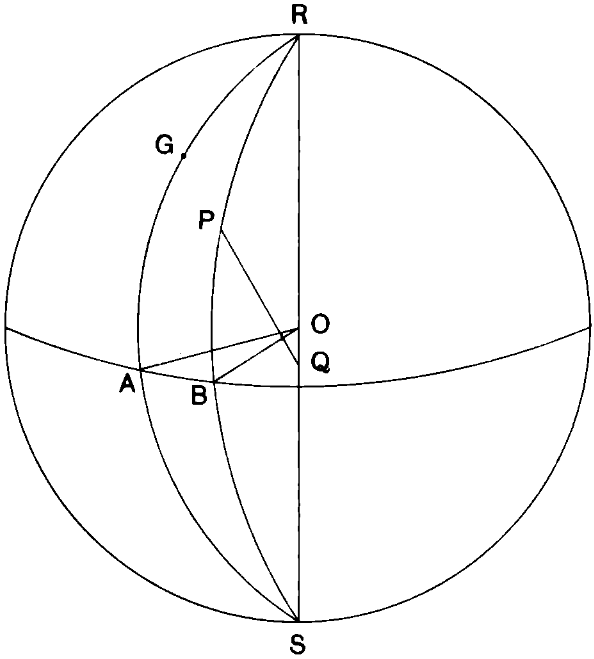

R
G
P
O
Q
A
B
S

Fig. 18

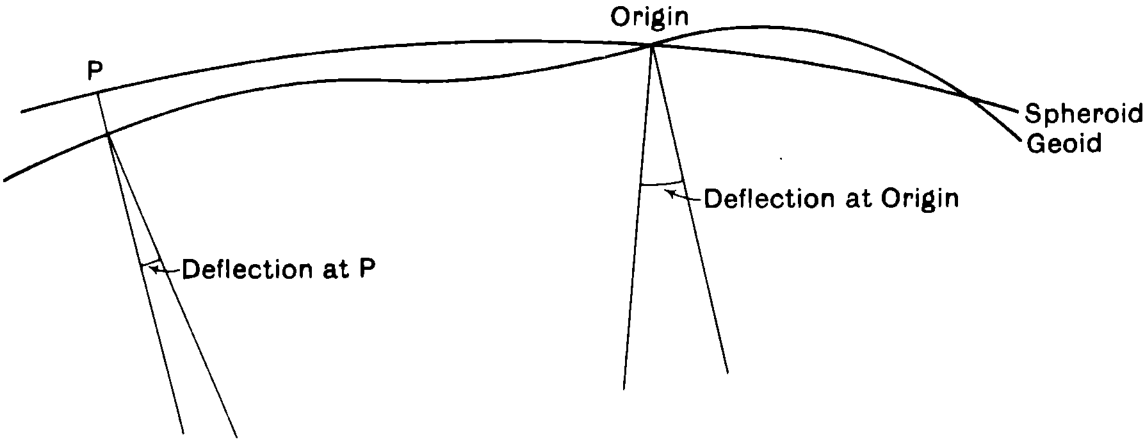
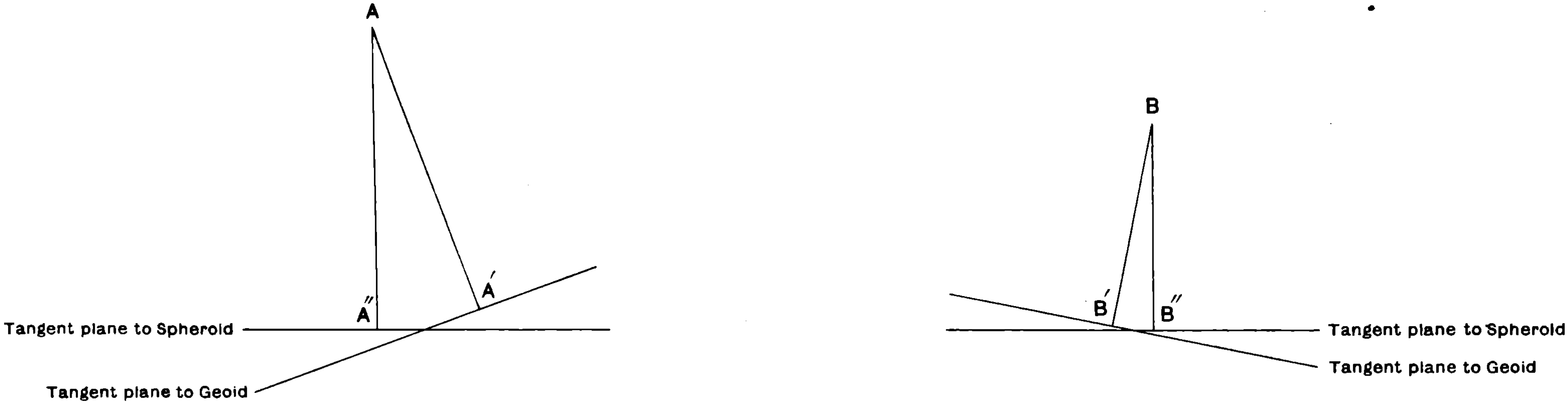

Origin
P
Spheroid
Geoid
Deflection at Origin
Deflection at P

Fig. 19

Fig. 20



Stations A & B as viewed from S

the figure that if the geoid and spheroid are inclined to each other, and if the points A and B are not exactly on the horizon, the angles $A' S B'$ and $A'' S B''$ are not equal. If a and β are the elevations of A and B, and if ζ and θ are the components of the deflection at right angles to SA and SB respectively, the required correction to the measured angle is $\zeta \tan a - \theta \tan \beta$. This correction is not negligible. In hilly country ζ may be $30''$, θ may be equal and of opposite sign if $(ASB = 180^\circ)$, a and β may each be elevations of 1°. Under these circumstances the resulting error will be $1''$. Such a large error is unusual, but it is possible that the error may be twice as much or even more*. It is to be remarked that the sum of the corrections to the three angles of a triangle is unlikely to be large, and that this source of error is not likely to result in bad triangular closures.

This correction can only be applied if the deflection is known. Unfortunately it is generally unknown, and the correction has never been applied†, but if the deflection could be determined to within $5''$, the possibility of serious error would be almost nil. The determination of the deflection in meridian to within 5 seconds can easily be done: it is only necessary to observe latitudes from one or two pairs of circum-meridian altitude stars. The determination of the deflection in the prime vertical is also readily determinable by a short azimuth programme (one observation of Polaris on each face on each of 10 zeros), provided the azimuth of the series is well controlled by Laplace Stations, and provided the latitude is not too low. South of about latitude 20°, it would be necessary to measure the deflection by longitude observations.

The worst errors could be avoided if such observations were made only at stations involving rays with exceptionally high elevations or depressions.

(*b*) *Height above sea-level.*—The latitude and longitude of a station are defined to be the co-ordinates of a point on the spheroid vertically below it. By a vertical line is meant the path which would be followed by a freely falling body if a well was dug in which it could fall. If the earth was a sphere, verticals would all be straight lines towards the centre of the earth: but since the earth is a spheroid, verticals are slightly curved lines, concave to the polar axis as shown in figure 21. This involves corrections to angles in two ways. Firstly the lines AA'' and BB'' of fig. 20 should have been drawn slightly

* For example consider the angle Banog H.S.— Dehra Dome Observatory T.S — Sirkanda H.S. in sheet 53 J. Here $\tan a$ and $\tan \theta$ are about $+ 1/10$ and $+ 1/12$ respectively. ζ and θ are about $+ 26''$ and $- 26''$ respectively, and the correction is $1\frac{1}{2}''$. This is a very extreme case.

† In 1931 it is being applied to the angles of the Kengtung Base net.

curved, and secondly there arises an additional dislevelment in correction (*a*) above, owing to the inclination of the elevated spheroidal level surface PQ to the spheroid P′Q′ (fig. 21)*. Both these corrections are very small. The first is the difference of two terms of the form:

$$0''\cdot 17\,(h'-h)\sin 2A\cos^2\lambda - 0''\cdot 14\,\frac{h'^2-h^2}{S}\sin A\sin 2\lambda,$$

where h is the height of the station, h' the height of the point observed, S the distance of the point, A its azimuth, and λ the latitude: h, h' and S being measured in miles. $h'-h$ will seldom be more than a mile, and the whole correction is unlikely to be as much as half a second.

The second correction is the difference of two terms of the form $0''\cdot 27 h \sin A \sin 2\lambda \tan a$, with the same notation as above. This will seldom be more than $0''\cdot 05$. When circumstances are such that either of these corrections is as large as the figures given above, the correction due to deflection is likely to be much larger †, and unless it can be applied, these others can also be neglected, especially the second.

(*c*) *Correction to Geodesic*, (see para 72).

Figure 23 illustrates a form of computation for these three corrections.

72. The spheroidal triangle.—The previous paragraph has shown how observed angles may be corrected to give the values which would have been obtained if the theodolite had been set up at sea-level with its axis normal to the spheroid.

If the reference figure was a sphere instead of a spheroid, the angle measured by the theodolite as corrected above would be the angle of the spherical triangle, i.e. of a triangle whose sides are great circles. On a spheroid great circles do not exist, and it is necessary to consider exactly what angle has been measured.

Let C be the station of observation and let stations A and B be observed with a theodolite. Let C′, A′ and B′ (fig. 22) be the points on the spheroid vertically below C, A and B. Then after correction as in para 71, the measured angle ACB is that between two planes containing A′ and B′ respectively and each containing the spheroidal normal at C′. These planes intersect the surface

* Deflection observations measure the angle between the spheroid and the actual level surface at the station of observation. For permanent record it is customary to correct them for the normal lack of parallelism between level surfaces at different heights. The correction now referred to merely amounts to removing these height corrections.

† Not necessarily in every angle, but there will certainly be some angles in which the deflection correction is much larger than these height corrections can ever be.

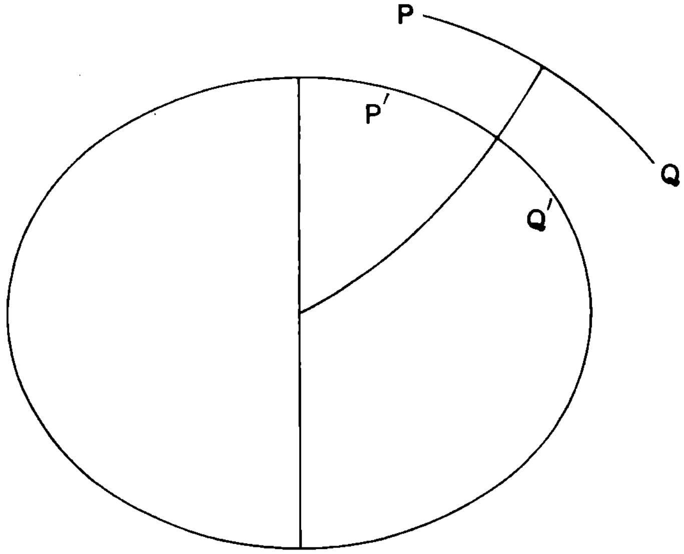


Fig. 21

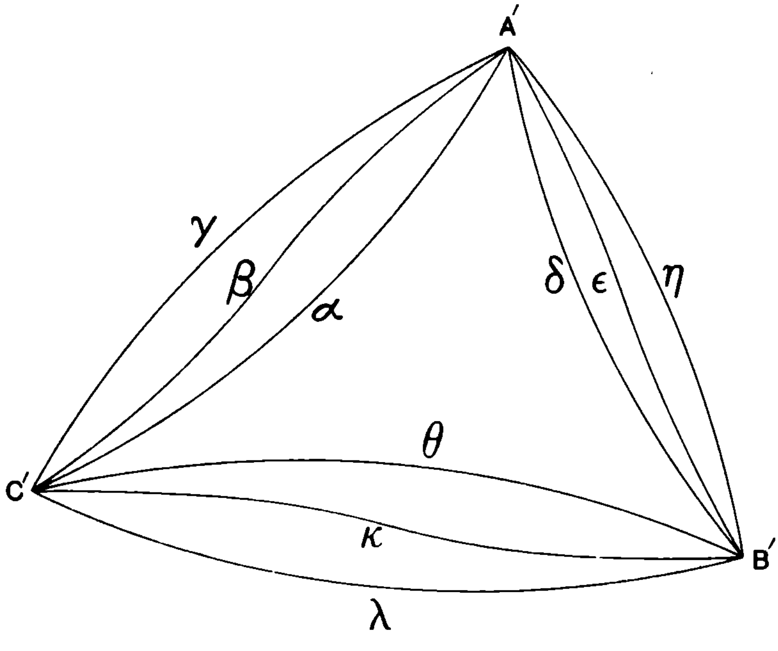


Fig. 22

Name of Series

Computation of Corrections to Observed Angles for deflection, altitude and geodesic.

N.B.—For correction to an observed angle ASB, A is the station that precedes B in the clockwise direction from S.

No.										
1	Deduction No.		1							
2	Station of Observation = S		Dehra Dun Obsy. T.S.							
3	Station A		Banog H.S							
4	Station B		Sirkanda New H.S							
5	Azimuth at S of A = A	of B = B	167° 36′	246° 55′	° ′	° ′	° ′	° ′	° ′	° ′
6	cos A	cos B	− 0·977	− 0·392	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
7	sin A	sin B	+ 0·215	− 0·920	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
8	Meridian Defln.* = η″	P.V. Defln.* = ξ″	− 37·0″	− 19·0″	− ″	− ″	− ″	− ″	− ″	− ″
9	ξ″ × cos A	ξ″ × cos B	+18·563	+ 7·488	− ·	− ·	− ·	− ·	− ·	− ·
10	η″ × sin A	η″ × sin B	− 7·955	+34·040	− ·	− ·	− ·	− ·	− ·	− ·
11	ξ″cos A − η″sin A = ζ″$_A$	ξ″cos B − η″sin B = ζ″$_B$	+26·518″	−26·552″	− ·	− ·	− ·	− ·	− ·	− ·
12	Obsd. altitude of A = α	Obsd. altitude of B = β	+ 5° 13′	− 4° 50′	−	−	−	−	−	−
13	tan α	tan β	+0·0914	+0·0846	−0·	−0·	−0·	−0·	−0·	−0·
14	(11) × (13) = ζ″$_A$ tan α	(11) × (13) = ζ″$_B$ tan β	+ 2·424	− 2·246	− ·	− ·	− ·	− ·	− ·	− ·
15	Deflection Corrn. = ζ″$_A$ tan α − ζ″$_B$ tan β		+ 4·670″		− ·		− ·		− ·	
16	Latitude of S = λ		30° 19′		° ′		° ′		° ′	
17	sin λ	cos λ	+0·505	+0·863	+0·	+0·	+0·	+0·	+0·	+0·
18	(Height of S) ÷ 1000 = h		2·255		·		·		·	
19	(Height of A) ÷ 1000 = h_a	(Height of B) ÷ 1000 = h_b	7·433	9·089	·	·	·	·	·	·
20	(19) − (18) = $h_a - h$	(19) − (18) = $h_b - h$	+ 5·178	+ 6·834	− ·	− ·	− ·	− ·	− ·	− ·
21	(19) + (18) = $h_a + h$	(19) + (18) = $h_b + h$	+ 9·688	+ 11·344	+ ·	+ ·	+ ·	+ ·	+ ·	+ ·
22	(6) × (7) = cos A sin A	(6) × (7) = cos B sin B	− 0·210	+ 0·361	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
23	cos² λ	cos² λ	+ 0·745	+ 0·745	+ 0·	+ 0·	+ 0·	+ 0·	+ 0·	+ 0·
24	(22) × (23) = ½sin2A cos²λ	(22) × (23) = ½sin2B cos²λ	− 0·156	+ 0·269	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
25	0·0644 × ($h_a - h$)	0·0644 × ($h_b - h$)	+ 0·334	+ 0·440	− ·	− ·	− ·	− ·	− ·	− ·
26	0·156 × ($h_a + h$)	0·156 × ($h_b + h$)	+ 1·511	+ 1·770	+ ·	+ ·	+ ·	+ ·	+ ·	+ ·
27	(25) × (26) ÷ $10^{-3}(h_a^2 - h^2)$	(25) × (26) ÷ $10^{-3}(h_b^2 - h^2)$	+0·505	+ 0·779	− ·	− ·	− ·	− ·	− ·	− ·
28	AS in miles	BS in miles	10·73	15·29	·	·	·	·	·	·
29	(27) ÷ (28) = $\frac{10^{-3}}{AS} \times (h_a^2 - h^2)$	(27) ÷ (28) = $\frac{10^{-3}}{BS} \times (h_b^2 - h^2)$	+ 0·047	+ 0·051	− ·	− ·	− ·	− ·	− ·	− ·
30	sin λ cos λ (line 17)	sin λ cos λ (line 17)	0·436	0·436	0·	0·	0·	0·	0·	0·
31	(7) × (30) = ½ sin A sin 2λ	(7) × (30) = ½ sin B sin 2λ	+0·094	−0·401	−0·	−0·	−0·	−0·	−0·	−0·
32	(24) × (25) = X_A	(24) × (25) = X_B	− 0·052″	+ 0·118″	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
33	$X_B - X_A$		+ 0·170″		− 0·		− 0·		− 0·	
34	(29) × (31) = Y_A	(29) × (31) = Y_B	+0·004″	−0·020″	−0·	−0·	−0·	−0·	−0·	−0·
35	$Y_B - Y_A$		− 0·024″		− 0·0		− 0·0		− 0·0	
36	First corrn. for altitude = (33) + (35)		+ 0·146″		− 0·		− 0·		− 0·	
37	(13) × (18) = h tan α	(13) × (18) = h tan β	+0·206	+0·191	− ·	− ·	− ·	− ·	− ·	− ·
38	$\frac{h}{10}$ tan α	$\frac{h}{10}$ tan β	+0·0206	+0·0191	−0·	−0·	−0·	−0·	−0·	−0·
39	(31) × (38) = Z_A	(31) × (38) = Z_B	+0·002″	−0·008″	−0·0	−0·0	−0·0	−0·0	−0·0	−0·0
40	Second corrn. for altitude = $Z_B - Z_A$		− 0·010″		− 0·		− 0·		− 0·	
41	− 0·14 (AS)² (AS from line 28)	0·14 (BS)² (BS from line 28)	− 16·1	− 32·8	− ·	− ·	− ·	− ·	− ·	− ·
42	(24) × (41) × 10^{-4} = G_A	(24) × (41) × 10^{-4} = G_B	+0·000″	−0·001″	−0·0	−0·0	−0·0	−0·0	−0·0	−0·0
43	Correction to geodesic = $G_B - G_A$		− 0·001″		− 0·0		− 0·0		− 0·0	
44	(15) + (36) + (40) + (43) = Total Correction		+ 4·805″		− ·		− ·		− ·	

* A minus sign indicates an easterly or northerly deflection of the plumb-line.

Computed by C. B. M. *Date* 18. 11. 31. *Checked by* G. P. R.

G.B.P.O.—1931.

of the spheroid in two curved lines C′ *a* A′ and C′ θ B′. Similarly, if observations are made at A, the angle obtained is that between two planes which intersect the spheroid in lines A′ γ C′ and A′ η B′: and it happens that A′ γ C′ and C′ *a* A′ do not exactly coincide, so that we have some of the angles of the six-sided figure shown in fig. 22. Such a figure cannot be described as a triangle. Before the computations can be proceeded with, it is necessary to choose some more or less arbitrarily defined line, passing through A and C, which can be called a side of the spheroidal triangle A′ C′ B′. A convenient line, and one whose geometrical properties are well known, is the "Geodesic" or the shortest line (on the surface) between A′ and C′. This line, which is shown as C β A′ in fig. 22, lies between C′ *a* A′ and C′ γ A′, and formulæ are known which give the angles at C′ between the tangents to these three lines. Consequently, given the angle *a* C′θ, it is possible to calculate β C′ κ.

The correction is the difference of two terms of the form:

$$0''{\cdot}07\left(\frac{S}{100}\right)^2 \cos^2\lambda \sin 2A,$$

where S is the side length in miles, A is the azimuth and λ is the latitude. With sides of less than 50 miles it may be safely neglected, and in view of the omission of the corrections of para 71, it may be neglected under all circumstances. If required, it is conveniently included in the form of computation given in fig. 23.

73. Solution of triangles.—Paras 71 and 72 have shown how the observed angles can be corrected to give the angles of the spheroidal triangle. Given these angles and the length of one side, it is required to obtain the length of the other two sides. Firstly let I, II and III be the angles of a spheroidal triangle, and let I_s, II_s and III_s be the angles of a spherical triangle whose sides are equal to those of the spheroidal triangle, the radius of the sphere being equal to K*, a mean radius of curvature of the spheroid in the latitude of the three stations concerned: then it can be shown that I_s = I etc. with an error of less than $0''{\cdot}001$ if sides are less than 100 miles long.

Again let I_p, II_p and III_p be the angles of a plane triangle whose sides are equal to those of this spherical triangle. Then it can be shown that $I_s - I_p = \frac{\epsilon}{3}$, etc. where ϵ is the spherical excess of the triangle. So that $I_p = I - \frac{\epsilon}{3}$ with sufficient accuracy, and

* K is given by $\frac{1}{K^2} = \frac{1}{3}\left(\frac{1}{K_1^2} + \frac{1}{K_2^2} + \frac{1}{K_3^2}\right)$, where $K_1 = \sqrt{\rho_1 \nu_1}$ etc., ρ_1 and ν_1 being the principal radii of curvature at station I.

if one side S_1 is known, the other sides may be obtained by the ordinary formula:

$$\frac{S_1}{\sin I_p} = \frac{S_2}{\sin II_p} = \frac{S_3}{\sin III_p}.$$

The spherical excess of a triangle is a small quantity which depends only on the area of the triangle (Δ) and on K, viz. $\epsilon = \frac{\Delta}{K^2}$. K depends only on the latitude, and is readily tabulated.

Before the observed angles are used for the solution of triangles, they are made to satisfy certain geometrical conditions, as described in the next paragraph.

74. Figural adjustment.—The angles of any figure are subject to certain geometrical conditions: for instance the sum of the angles of a plane triangle is 180°. Since no observations are perfect, the observed * angles will not exactly satisfy these conditions, and it is possible to obtain improved values of the angles by distributing among them such minimum corrections as will cause them to satisfy the conditions. There are three kinds of figural conditions, viz.:—

(*a*) Triangular conditions.
(*b*) Side conditions.
(*c*) Central conditions.

A triangular condition is simply that the sum of the three angles of a spherical triangle is equal to 180° *plus* the spherical excess.

A side condition occurs when it is possible to compute the ratio between two sides of a figure by different routes, using some independent angles in each deduction: the ratios obtained by each route should of course be identical.

A central condition is that, if the angles observed at any point comprise the whole horizon, their sum must be 360°. Such conditions occur only in centred figures, not in braced quadrilaterals.

A single triangle, not forming part of a more complicated figure contains only one condition, a triangular condition. It is satisfied by distributing the triangular error among the angles in inverse proportion to the weights of the three observed angles (see para 75).

A braced quadrilateral contains three triangular conditions and one side condition: it of course contains four triangles, but the four triangular errors are not independent: given the errors of three of the triangles that of the fourth is immediately known, and if the three conditions are satisfied the fourth will also be, automatically. Similarly in a quadrilateral ABCD the ratio of CD to AB can be

* Ideally these observed angles should first be corrected as in paras 71 and 72.

obtained through the triangles ACB and ACD with AC as a common side, or through three other pairs of triangles with BD, AD and CB as common sides respectively: but if any two of these routes give accordant results, the remaining two will necessarily agree with them, and there is only one condition.

A single-centred quadrilateral or other polygon contains as many triangular conditions as the number of sides, (i.e. 4 for a quadrilateral, 5 for a pentagon, etc.), one central condition and one side condition.

The solution for the corrections to the angles is made by the usual process of "least squares" and is generally known as "grinding". Regular computation forms are provided for the adjustment of braced quadrilaterals (form 9 Trian.), and of centred quadrilaterals, pentagons or hexagons (form 10 Trian.). The theory on which these forms are based is given in Eccles' "Notes on the Theory of Errors of Observation"; and also in G.T. Vol. II, pages 104-110.

It sometimes happens that more complicated figures are observed, such as a base net, or two interlocking polygons. In such a case the form of computation has to be improvised. Rules are given in Eccles' "Notes", and in G.T. Vol. II, pages 210-217 and 225-238, where two examples are also given.

A fourth kind of condition between the observed angles is that known as a "toto-partial" condition, namely that the value of two or more adjacent angles at a station must be equal to the value of their sum. If at the time of observation all stations are visible, whenever required, and if the procedure given in para 48 is always followed, these conditions will always be automatically fulfilled, without consideration. When broken rounds have to be observed, some care is required, but if the abstract is prepared in the manner described in para 56, the toto-partial equations will always be satisfied and will require no consideration.

75. Weights.—The weight of an angle is a measure of its reliability: it is inversely proportional to the square of its probable error*. In order to compute the proper distribution of the misclosures of the various condition equations it is necessary to know the weights of the observed values of each angle. Weights can only be determined on the assumption that errors follow the laws of probability, but actually various unsuspected systematic errors may have a much more serious effect than ordinary casual errors of intersection and graduation. Consequently, no amount of

* The weight is sometimes defined to be equal to the reciprocal of the square of the root-mean-square error. This is of course proportional to the reciprocal of the square of the probable error. The weights recorded in Survey of India computations are really the reciprocals of the square of the root-mean-square error.

computation will serve to obtain a reliable value of the weight. An accuracy of 10% would be ample, if it would be obtained, but there is no guarantee that the weights obtained by any method are as reliable as this: it is even possible that they may sometimes be actually misleading.

The most usual method of determining the average probable error of the angles of a series is to derive it from the triangular errors, but this method does not serve to determine the relative probable errors of the separate angles of the same figure, which is what is now required. The best and easiest criterion available for this purpose is the agreement between the mean values obtained on different zeros: an angle, in which the different zero means all agree within 3″ may fairly be considered more reliable than one in which they range through 6″. But before this criterion can be applied it is necessary to correct the various zero means for any systematic graduation error of the theodolite. In many theodolites there is systematic graduation error in different parts of the circle: measures made in two of the quadrants may tend to be always too high, and measures made in the other two may always be too low; such a regular graduation error is a source of very little inaccuracy if measures are made on 10 zeros: the errors of the different zeros tend to positive cancellation, and the probable error of the general mean is not merely $\sqrt{10}$ times as small as that of a single zero mean. In such a theodolite, the zero mean of the most perfectly observed angle may range through 5″, and the general mean will be very good. While in a carelessly observed angle the range may be no more than 8″, and the mean will be bad: its probable error will be much more than 8/5 times as much as that of the good angle.

On completion of each season's work the horizontal angles of each theodolite should be analysed for systematic graduation error (see para 78). This involves some labour, but it is a useful end in itself. The various zero means of each angle are then corrected for this graduation error, and the agreement of the corrected zero means *inter se* is the guide to the probable error of the angle.

76. Computation of co-ordinates.—The figures having been ground, the sides of the triangles are computed as in para 73. The next step is the computation of the co-ordinates of the stations.

Suppose the latitude and longitude of A′ and B′ (fig. 22) to be already known: then given the azimuth at A′ of B′, the angle at A′ between the geodesics A′ B′ and A′ C′, and the length of the geodesic A′ C′, it is possible to calculate the latitude and longitude of C′. (By the azimuth of B′ at A′ is meant the angle at A′ between the tangent to the spheroidal meridian and the tangent to the geodesic A′ B′). And given similar data at B′, the co-ordinates of C′ can again be deduced in the same way. Several formulæ exist for this purpose. That which has hitherto been used in the Survey of India is due to Puissant, and is described in G.T. Vol. II, pages 112-118. The formula which has now been adopted is that given by Dr. J. de Graaff Hunter*. It has the advantage that a slip in the early part of the computation does not vitiate every succeeding line, as is the case with the old form.

It is to be noticed that the required angle C′ A′ B′ is the observed angle corrected for figural adjustment †, but not corrected for spherical excess. The spheroidal angle is required, not the angle of a hypothetical plane triangle. Provision for obtaining the correct angle is made on the form 13A Trian.

77. Heights.—The height of any station is defined to be the vertical separation between it and the geoid beneath it. Given the distance and elevation of a station B as seen from a station A of known height, the computation of the height of B would be a straightforward matter, but for two difficulties, namely refraction and the fact that the shape of the geoid is not well known, so that the correction for curvature cannot be properly computed.

Refraction.—Light passing through air (whose density varies with height) is curved in a vertical plane. The extent to which it is curved depends on the rate of change of the air's density with height, which is greater at low altitudes than at high, and which is liable to be abnormally large or small in the neighbourhood of the ground. The amount of curvature is usually expressed as a "Coefficient of refraction" k, which is equal to half that fraction of a second through which the light is deviated while passing through a distance which subtends one second at the earth's centre. Apart from atmospheric abnormalities the normal (minimum) value of k varies with altitude and also with the temperature and pressure. It is given in Table 5 Sur. of the Auxiliary Tables, 5th edition.

If the coefficient of refraction was constant throughout the length of a ray the proper correction to an observed vertical angle would be $-k \times$ (Distance expressed in seconds of arc), but when a ray proceeds from one height to another, the coefficient is not expected to be constant: a suitable mean value is then obtained by abstracting from the table the value of k appropriate to the height of the ray at a point distant one third of its length from the station of observation. (See examples in the heading of the table).

It is important to note that the tabular value of k is only applicable to observations made at the time of minimum refraction,

* "*Geodesy*", Departmental Paper No. 12, pages 35-37.

† Also for deviation of the vertical, height, and geodesic, if possible.

and also that it cannot be expected to approach the truth in the case of low grazing rays.

If vertical angles are observed at both ends of a ray, and if the coefficient of refraction is constant throughout the length of the ray, and if it is also constant at the times when both observations are made, any departure of this constant coefficient from the tabular value will result in equal and opposite errors in the value of the height difference deduced from the two ends of the ray, and the mean will consequently be correct. This is the basis of the well-known method of computation by "reciprocal vertical angles". If the ray covers a wide range of altitude, the result will be badly in error, since the coefficient of refraction is definitely not constant throughout the length of the ray. On the other hand it may well happen that the actual air density gradient differs from that which was assumed in the construction of Table 5 Sur., in which case computation based on that table will also be incorrect. But, given normal conditions, the difference between the actual and assumed gradients may be expected to be approximately equal throughout the length of the ray, with the result that a combination of the two methods will give a result which is better than either.

Thus, the difference of height should always be computed from both ends, and the mean accepted. The difference between the two results will sometimes be seen to be systematic (e.g. the difference deduced from the observation at the higher end may always be greater than that from the lower): in such a case the discrepancy may be expected to vary as the square of the length of the ray.

If observations at one end of the ray are lacking, or if they are considered unreliable, a determination of height difference based on observation at the other end only cannot be considered satisfactory, unless reciprocal determinations have been giving accordant results elsewhere. If they have not, suitable allowance must be made; either by computing values of k from other rays, comparing the computed values with the tabular, and so deducing a suitable correction to the table; or by abstracting the discrepancies found in rays of different lengths, and so deducing an expected discrepancy in the ray in question (assuming discrepancy to vary as the square of the length), and so improvising a reverse value of the height difference to mean with that already computed in the regular manner. See Geodetic Report Vol. V, pages 87-88. This procedure must always be followed in the case of intersected points, unless the distances are so short as to make it unnecessary.

Curvature correction.—All surveyors are familiar with the correction to height on account of the Earth's curvature. If the Earth is, or can be assumed to be, a sphere or a spheroid, this correction presents no difficulty: but since heights above the geoid are required, and since the geoid is an irregular figure, the curvature correction is not altogether determinate. The usual procedure, which is generally adequate, is to assume the curvature of the geoid to be equal to that of the spheroid in the latitude and azimuth concerned. In so far as the difference between the radii of curvature of the geoid and the spheroid is constant between the two stations, the error is cancelled by meaning reciprocal observations, and generally this will not be a serious source of error among hills of ordinary magnitude and with rays averaging 20 miles in length.

But in the case of rays to Himālayan peaks, which may be 100 miles long, and which pass over country in which the geoid may be expected to be extremely irregular, this approximation may lead to results which are far from the truth. In such a case a better result can probably be obtained by first computing the relative heights of the two stations (or station and intersected point) above the spheroid, and then estimating the difference of the separations of geoid and spheroid at the two points. To compute heights above the spheroid it is necessary to correct the observed vertical angles for the lack of parallelism between geoid and spheroid: this can only be determined by astronomical observation of the deflection. To compute the separations between the geoid and the spheroid it is necessary to make some assumption (perhaps quite inaccurate) regarding the isostatic compensation of the hills. The computation of heights of distant Himālayan peaks will always need special consideration and details cannot be given here. Reference may be made to Geodetic Report Vol. I, Chapter VI and to Vol. III, Chapter VIII (2nd part). A table to facilitate the computation of geoidal rise on the assumption of perfect compensation is given in Geodetic Report Vol. V, page 79.

78. Routine of computation.—On return from the field the duplicate angle books and abstracts should be received back from the Director (see paras 55 and 56). Angle books and abstracts should normally be fully completed in the field, but if they have unavoidably fallen into arrears the first step is to complete them, thereby obtaining the final mean values of all the observed angles.

Data.—The Officer in charge of the Computing Office should be asked to supply the co-ordinates (to three decimals of a second) of any existing stations to which connections have been made, and also the lengths (to 7th decimal of log) and azimuths (to two decimals of a second) of the sides joining them. These should be compared with the triangulation pamphlet and any discrepancy enquired into.

Analysis of graduation error.—The next step is to analyse the horizontal circle readings for systematic graduation error, as a preliminary to computing the weights of the angles (see para 75). In the case of a theodolite which has done many years' work, and whose systematic graduation error has always been found to be negligible or sufficiently constant, it may be possible to omit this step every alternate year. But it gives useful information regarding the condition of the instrument and it should always be included if circumstances permit. If only a small programme has been observed (less than about 10 stations) it will not be possible to get a reliable value of the error from the season's work, and values obtained in previous years will have to be utilised. The analysis consists simply of classifying the observed angles according to the part of the circle on which they have been measured, and recording the values of the "Zero mean"* *minus* "General mean" as given in the Abstract form. The form in which the angles are classified is that of a double entry table (see fig. 24). Along the top of the table is recorded the part of the circle on which the left hand arm of the angle was measured by A microscope, and down the side of the table is the part of the circle on which the right hand arm was measured. The entries in the table are the values of ZM – GM as taken from the abstract, the total number of entries being the number of angles measured multiplied by the number of zeros* on which each angle was observed. Many of the squares in the table are inevitably blank since the angles of triangulation generally lie between about 30° and 90°: the entries congregate about a line passing diagonally (but eccentrically) across the table. When a square contains more than one entry the mean is taken out and recorded at the bottom of the square. Some care is required in entering values of ZM – GM in their correct squares, reference to the original angle book being necessary. The first three columns of form 7A Trian. may advantageously be completed before the classification is taken up. When the entry has been made for any one zero on which an angle has been measured, the entries corresponding to the remaining zeros can be entered without trouble, the correct square for each entry being one or two or more squares diagonally distant from the preceding square. To facilitate this process the classification interval should be equal to, or a submultiple of, the separation of the different zeros: if the zeros have been 0°, 18°, 36°, etc., the classification should be in groups of 9°, not of 15° as given in the example. It will seldom be convenient to make groups of less than 9° or more than 20°.

Any angles smaller than 15° (whatever may be the size of the groups in the table), or greater than 165°, should be excluded. Their variations cannot be attributed to systematic graduation error. Obviously no entries may ever be made in the centre diagonal, corresponding to angles of zero size.

* In the case of a three-microscope theodolite the zero means are abstracted separately for FR and FL and the two faces are treated as separate zeros throughout these operations.

Fig. 24.

Classification of angles measured with 12-inch Theodolite No. II on Mong Hsat Series, 1929-30.

Values of ZM−GM, where ZM is the mean value of all measures of an angle taken on a certain part of the circle, and GM is the mean of all the measures on all parts of the circle.

Left Arm / Right Arm	180°–195° 0–15	195°–210° 15–30	210°–225° 30–45	225°–240° 45–60	240°–255° 60–75	255°–270° 75–90	270°–285° 90–105	285°–300° 105–120	300°–315° 120–135	315°–330° 135–150	330°–345° 150–165	345°–360° 165–180
180°–195° 0–15								− 0·1 − 4·5 + 0·3 − 0·3 S = − 4·5 M = − 1·1	− 0·6 − 2·5 − 1·1 − 1·7 − 1·9 + 0·5 − 3·3 S = − 10·6 M = − 1·5	+ 0·1 − 0·9 + 0·4 − 3·5 + 0·3 + 0·5 − 0·7 + 1·3 − 0·8 − 2·8 + 1·1 S = − 5·0 M = − 0·5	+ 0·1 − 0·7 − 1·0 + 0·8 − 0·6 − 0·3 + 0·3 S = − 1·4 M = − 0·2	
195–210 15–30									+ 0·8 − 2·6 − 1·3 − 1·0 S = − 4·1 M = − 1·0	− 1·1 − 2·6 − 1·6 − 1·3 + 1·1 − 0·4 − 2·1 S = − 8·0 M = − 1·1	+ 1·1 + 1·0 + 0·8 − 0·3 − 1·3 + 1·5 + 3·3 + 0·1 − 1·3 + 0·5 S = + 5·0 M = + 0·5	− 0·2 + 0·7 + 0·7 + 1·2 + 0·8 − 0·0 + 0·6 S = + 3·8 M = + 0·5
210–225 30–45	− 0·8 + 1·7 − 1·9 + 0·8 − 0·1 + 2·7 S = + 2·4 M = + 0·4									− 4·0 + 1·6 + 0·5 − 1·3 S = − 3·2 M = − 0·8	− 0·7 − 0·5 + 1·3 − 0·8 + 0·9 + 3·3 + 0·1 + 0·4 S = + 4·0 M = + 0·5	+ 0·8 + 0·2 − 0·1 + 0·9 + 1·0 − 0·5 − 0·0 − 1·0 − 0·0 − 2·4 − 0·5 + 0·6 S = − 0·8 M = − 0·1
225–240 45–60	+ 1·3 + 0·1 + 1·6 + 0·6 + 1·1 + 1·4 + 0·1 + 1·3 + 1·3 + 1·7 + 1·9 S = + 12·4 M = + 1·1	− 0·4 − 1·5 + 1·7 − 0·6 − 0·3 + 1·7 S = + 0·6 M = + 0·1									− 0·5 + 3·5 − 1·0 + 0·4 S = + 2·4 M = + 0·6	− 0·8 − 0·0 + 2·7 + 1·9 + 0·3 + 2·0 + 1·5 − 2·6 S = + 5·0 M = + 0·6
240–255 60–75	− 1·2 − 2·8 + 1·3 + 0·9 − 0·4 + 2·8 + 0·5 + 2·0 S = + 3·1 M = + 0·4	+ 2·2 − 0·6 + 0·4 − 0·9 + 3·1 − 0·2 + 3·3 − 1·2 + 0·8 + 2·0 + 2·4 − 0·7 S = + 10·6 M = + 0·9	+ 1·5 + 1·3 + 1·5 − 1·2 + 1·3 S = + 4·4 M = + 0·9									+ 1·3 + 0·5 + 0·5 + 1·9 S = + 4·2 M = + 1·1
255–270 75–90	− 0·4 − 0·1 − 1·7 − 2·7 S = − 4·9 M = − 1·2	− 0·5 − 0·0 − 1·0 − 1·5 + 0·8 − 0·9 + 1·5 + 1·1 S = − 0·5 M = − 0·1	− 1·1 + 0·1 + 1·4 + 0·4 − 2·1 − 0·2 − 0·6 + 1·5 − 1·7 − 0·1 − 0·6 + 1·5 S = − 1·5 M = − 0·1	− 2·5 − 1·3 − 0·9 − 2·1 − 1·8 S = − 8·6 M = − 1·7								
270–285 90–105		+ 1·4 + 1·9 − 1·1 − 2·5 S = − 0·3 M = − 0·1	− 2·1 + 2·0 − 2·2 + 0·2 − 0·5 − 1·5 + 0·4 + 1·1 S = − 2·6 M = − 0·3	− 2·2 − 0·3 + 0·6 − 2·1 − 0·1 − 1·8 − 1·6 − 1·8 − 0·9 + 0·6 − 0·6 S = − 10·2 M = − 0·9	− 1·8 − 0·1 + 1·1 − 1·2 − 2·4 − 2·5 S = − 6·9 M = − 1·2							
285–300 105–120			+ 2·6 + 3·8 + 3·3 + 2·8 S = + 12·5 M = + 3·1	− 1·9 + 1·8 + 0·9 + 2·2 − 0·4 − 2·2 − 0·3 + 4·5 S = + 4·6 M = + 0·6	+ 1·4 − 0·3 + 2·1 + 0·9 − 1·7 + 0·4 + 0·3 − 1·9 + 0·9 − 1·5 + 1·9 S = + 2·5 M = + 0·2	+ 5·9 + 1·9 + 1·0 + 3·2 − 0·0 − 0·1 S = + 11·9 M = + 2·0						
300–315 120–135				+ 1·5 + 0·8 + 1·0 + 1·8 S = + 5·1 M = + 1·3	+ 2·7 + 1·0 + 1·8 + 0·3 + 2·3 − 0·7 + 0·3 + 1·6 S = + 9·3 M = + 1·2	− 0·4 + 0·5 + 3·7 − 0·3 + 0·7 + 3·9 + 0·7 + 4·0 − 0·2 + 3·3 S = + 15·9 M = + 1·6	+ 1·8 + 3·0 − 0·7 + 0·5 + 2·6 + 2·0 − 0·8 S = + 8·4 M = + 1·2					
315–330 135–150					− 1·0 − 1·8 − 1·3 − 1·4 S = − 5·5 M = − 1·4	+ 4·8 + 1·0 + 3·0 + 0·9 + 0·9 + 0·4 − 0·4 − 0·5 S = + 10·1 M = + 1·3	+ 0·8 + 1·6 + 0·4 + 2·7 − 1·2 + 2·4 + 1·4 − 0·3 + 2·7 + 2·6 S = + 13·5 M = + 1·4	+ 0·1 + 0·8 + 0·3 − 1·7 + 0·7 + 1·3 + 0·2 S = + 1·6 M = + 0·2				
330–345 150–165						− 1·0 − 1·2 + 0·4 + 0·9 S = − 0·9 M = − 0·2	+ 1·6 − 0·5 − 0·6 − 0·4 − 1·5 + 0·1 − 0·4 − 1·3 S = − 3·0 M = − 0·4	− 0·4 − 3·4 − 2·2 + 0·2 − 0·9 − 2·3 − 2·8 − 0·5 − 0·9 − 1·2 S = − 14·4 M = − 1·4	− 2·3 − 4·1 + 0·4 − 2·1 − 2·7 + 1·1 − 1·5 S = − 11·2 M = − 1·6			
345–360 165–180							− 1·5 − 1·8 + 0·4 + 1·8 S = − 1·1 M = − 0·3	+ 1·2 − 2·2 − 1·4 + 0·4 − 2·0 − 3·1 − 0·9 S = − 8·0 M = − 1·1	+ 0·0 − 2·3 − 2·5 − 3·0 − 0·7 − 2·4 − 0·9 − 2·2 − 1·2 − 1·9 − 1·1 S = − 18·6 M = − 1·7	− 1·7 − 2·6 − 0·8 − 0·5 − 2·7 + 0·2 − 0·5 S = − 8·6 M = − 1·2		

Reg No 443 M.D. 31- 400

H.S.I.O. Dehra Dun.

Analysis of Graduation Error of 12-inch Theodolite No. II.

a=mean from corresponding square on Fig. 24 (classification of angles), b = number of entries in the corresponding square on Fig. 24, c=(a×b), d = mean angle, e= 15×c/d, f = mean scale reading.

Left Arm → / Right Arm ↓		180°–195° 0–15	195°–210° 15–30	210°–225° 30–45	225°–240° 45–60	240°–255° 60–75	255°–270° 75–90	270°–285° 90–105	285°–300° 105–120	300°–315° 120–135	315°–330° 135–150	330°–345° 150–165	345°–360° 165–180
180°–195° 0–15	a= b= c= d= e= f=								−1·1 4 −4·4 75 −0·88 150	−1·5 7 −10·5 60 −2·63 157·5	−0·5 11 −5·5 45 −1·83 165	−0·2 7 −1·4 30 −0·70 172·5	
195–210 15–30	a= b= c= d= e= f=									−1·0 4 −4·0 75 −0·80 165	−1·1 7 −7·7 60 −1·93 172·5	+0·5 10 +5·0 45 +1·67 180	+0·5 7 +3·5 30 +1·75 7·5
210–225 30–45	a= b= c= d= e= f=	+0·4 6 +2·4 30 +1·20 22·5									−0·8 4 −3·2 75 −0·64 180	+0·5 8 +4·0 60 +1·00 7·5	−0·1 12 −1·2 45 −0·40 15
225–240 45–60	a= b= c= d= e= f=	+1·1 11 +12·1 45 +4·03 30	+0·1 6 +0·6 30 +0·30 37·5									+0·6 4 +2·4 75 +0·48 15	+0·6 8 +4·8 60 +1·20 22·5
240–255 60–75	a= b= c= d= e= f=	+0·4 8 +3·2 60 +0·80 37·5	+0·9 12 +10·8 45 +3·60 45	+0·9 5 +4·5 30 +2·25 52·5									+1·1 4 +4·4 75 +0·88 30
255–270 75–90	a= b= c= d= e= f=	1·2 4 −4·8 75 −0·96 45	−0·1 8 −0·8 60 −0·20 52·5	−0·1 12 −1·2 45 −0·40 60	−1·7 5 −8·5 30 −4·25 67·5								
270–285 90–105	a= b= c= d= e= f=		−0·1 4 −0·4 75 −0·08 60	−0·3 8 −2·4 60 −0·60 67·5	−0·9 11 −9·9 45 −3·30 75	−1·2 6 −7·2 30 −3·60 82·5							
285–300 105–120	a= b= c= d= e= f=			+3·1 4 +12·4 75 +2·48 75	+0·6 8 +4·8 60 +1·20 82·5	+0·2 11 +2·2 45 +0·73 90	+2·0 6 +12·0 30 +6·00 97·5						
300–315 120–135	a= b= c= d= e= f=				+1·3 4 +5·2 75 +1·04 90	+1·2 8 +9·6 60 +2·40 97·5	+1·6 10 +16·0 45 +5·33 105	+1·2 7 +8·4 30 +4·20 112·5					
315–330 135–150	a= b= c= d= e= f=					−1·4 4 −5·6 75 −1·12 105	+1·3 8 +10·4 60 +2·60 112·5	+1·4 10 +14·0 45 +4·67 120	+0·2 7 +1·4 30 +0·70 127·5				
330–345 150–165	a= b= c= d= e= f=						−0·2 4 −0·8 75 −0·16 120	−0·4 8 −3·2 60 −0·80 127·5	−1·4 10 −14·0 45 −4·67 135	−1·6 7 −11·2 30 −5·60 142·5			
345–360 165–180	a= b= c= d= e= f=							−0·3 4 −1·2 75 −0·24 135	−1·1 7 −7·7 60 −1·93 142·5	−1·7 11 −18·7 45 −6·23 150	−1·2 7 −8·4 30 −4·20 157·5		

The table illustrated is appropriate to a two-microscope theodolite. On such a theodolite, if A microscope reads (e.g.) 60°, the parts of the circle in use by the two microscopes are identically the same as when A reads 240°, consequently separate lines and columns are not provided in the table for circle readings of greater than 180°. On a two-microscope theodolite FL and FR readings are also on identically the same part of the circle. For the analysis of a three-microscope theodolite, independent columns will only be provided up to 120°, along the top and bottom, and when abstracting values of ZM – GM, face left and face right readings must be treated separately.

If sufficient material was available the mean of all the entries in each compartment of the table would be all that is necessary for the correction of observed angles for the deduction of weights, but since graduation error is not the only source of discrepancy between zero means and general means, and since material is generally too scanty to average out other sources of error satisfactorily, it is necessary to smooth the results. To do this the means * are abstracted to a further table (fig. 25) with the same number of rows and columns as the preceding table. Each square of this table contains six entries one below the other. The first (a) is the mean value of the corresponding square in the previous table, e.g. + 0·5 for the 11th column of the second row in the example. The second (b) is the number of entries of which this figure is the mean, e.g. 10. The third (c)* is the product of (a) and (b), viz. 5·0. The fourth (d) is the mean size of the angle to which the square refers, e.g. 195° – 150° = 45°. (The angles in the column heading should be subtracted from those entered on the left of each row, the pairs selected being such that the difference is positive and less than 180°). The fifth entry (e) is $k \times (c) \div (d)$, where k is the graduation interval between successive lines and columns e.g. 15°. In this example (e) is 1·67, the significance of which is that the mean rate (multiplied by its weight) at which graduation error is changing in the part of the circle under consideration is + 1·″67 (seconds) per 15°. The last entry (f) is the middle scale reading of this part of the circle, viz. $\frac{1}{2}$ (202½ + 157½) = 180°. The figure 1·″67 (divided by its weight—in this example 10) per 15° is then applicable to the circle at 180°.

It will be noticed that equal values of (f) run diagonally across the table. The next step is to sum all the entries (e) and (b) in all the squares containing any one value of (f) : this will sometimes involve reference to two diagonal sets of squares. The sum of all the entries (e) is then divided by the sum of the entries (b)

* In practice it is more convenient to write down the totals instead of the means in the first table, and to copy them direct into entry (c) of the second table, entries (a) being superfluous.

and the result is the weighted mean rate of change of graduation error of the circle in the neighbourhood of (f). The actual graduation error of the circle is obtained by forming the running total of the halves* of these quotients, each item of the running total being considered to be the graduation error at a part of the circle whose reading is K°/4 greater than the (f) corresponding to the quotient last added.

It will be convenient to plot this running total in the form of a curve (see fig. 26).

Weights.—The weights of the observed angles are computed on form 7A Trian.† (see fig. 27). Each angle requires as many lines as the number of zeros which have been observed. (Twice as many in the case of a three-micrometer theodolite). In the first two columns are entered (to the nearest degree) the part of the circle on which A micrometer has been read, (the entry in the first column being the circle reading of the left hand arm i.e. the smaller reading unless the 360° graduation happens to intervene). In the third column is entered the value of ZM *minus* GM. In the fourth column is recorded the graduation error (from the running total or curve) corresponding to the entry in the first column, and in the 5th is the graduation error corresponding to the second column. The sixth column records (3) + (4) – (5). Columns (3), (4), (5) and (6) should be recorded to the nearest tenth of a second. The probable error of the angle is then obtained from the entries in the sixth column by the formula.:—

$$p.e. = \frac{0{\cdot}845\, \Sigma |v|}{n\sqrt{n-1}},$$

where $\Sigma |v|$ is the sum of column (6), without regard to sign, and n is the number of entries. The weight usually recorded is the reciprocal of the square of the root-mean-square error, viz.:— the reciprocal of 0·454 × (probable error)^2.

If the weight of a double angle (the sum of two or more adjacent angles) is required, the columns 6 of the form 7A Trian. of each of the partial angles should be added together algebraically,

* Halves because entries occur at intervals of K°/2 whereas the quotient expresses the rate of change in seconds per K°.

† The form 7 Trian. in which weights have hitherto been computed allows for a rather elaborate computation of errors due to observation and graduation respectively, but no allowance is made for the almost absolute cancellation of regularly systematic graduation error. If such systematic error is present, the relative weights obtained are fallacious, while if graduation error is absent, it is thought that a sufficiently reliable result is obtainable from a simple consideration of discrepancies between zero means

Similarly it is now thought unsound to apply a correction to the general mean, if observations on some zeros are more consistent than those on others, as this will give undue weight to the graduation error of the apparently better zeros. If observations on any zero are exceptionally inconsistent, the weight of that zero is improved by repeat observations (see para 58).

and the result is the weighted mean rate of change of graduation error of the circle in the neighbourhood of (f). The actual graduation error of the circle is obtained by forming the running total of the halves* of these quotients, each item of the running total being considered to be the graduation error at a part of the circle whose reading is K°/4 greater than the (f) corresponding to the quotient last added.

It will be convenient to plot this running total in the form of a curve (see fig. 26).

Weights.—The weights of the observed angles are computed on form 7A Trian.† (see fig. 27). Each angle requires as many lines as the number of zeros which have been observed. (Twice as many in the case of a three-micrometer theodolite). In the first two columns are entered (to the nearest degree) the part of the circle on which A micrometer has been read, (the entry in the first column being the circle reading of the left hand arm i.e. the smaller reading unless the 360° graduation happens to intervene). In the third column is entered the value of ZM *minus* GM. In the fourth column is recorded the graduation error (from the running total or curve) corresponding to the entry in the first column, and in the 5th is the graduation error corresponding to the second column. The sixth column records (3) + (4) − (5). Columns (3), (4), (5) and (6) should be recorded to the nearest tenth of a second. The probable error of the angle is then obtained from the entries in the sixth column by the formula.:—

$$p.e. = \frac{0{\cdot}845\,\Sigma|v|}{n\sqrt{n-1}},$$

where $\Sigma|v|$ is the sum of column (6), without regard to sign, and n is the number of entries. The weight usually recorded is the reciprocal of the square of the root-mean-square error, viz.:—the reciprocal of $0{\cdot}454 \times (\text{probable error})^2$.

If the weight of a double angle (the sum of two or more adjacent angles) is required, the columns 6 of the form 7A Trian. of each of the partial angles should be added together algebraically,

* Halves because entries occur at intervals of K°/2 whereas the quotient expresses the rate of change in seconds per K°.

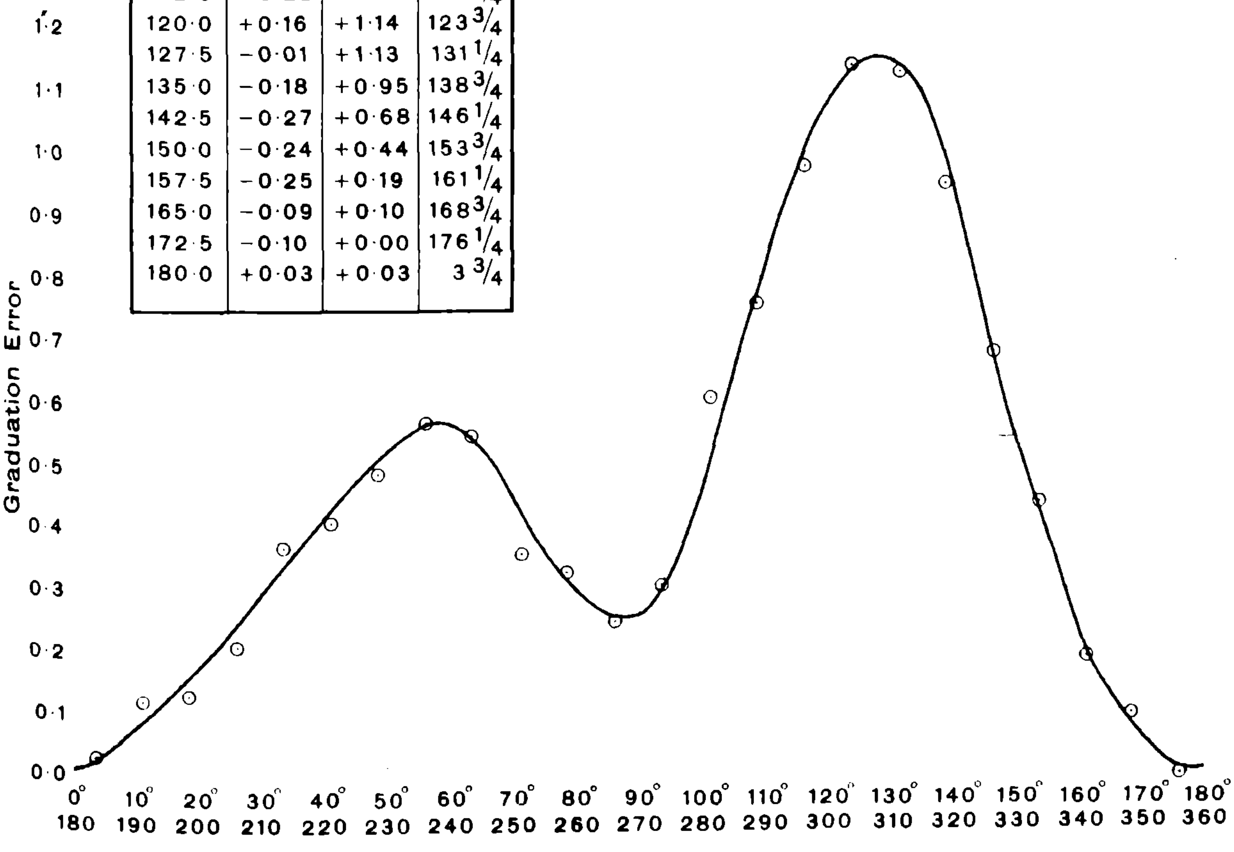
† The form 7 Trian. in which weights have hitherto been computed allows for a rather elaborate computation of errors due to observation and graduation respectively, but no allowance is made for the almost absolute cancellation of regularly systematic graduation error. If such systematic error is present, the relative weights obtained are fallacious, while if graduation error is absent, it is thought that a sufficiently reliable result is obtainable from a simple consideration of discrepancies between zero means

Similarly it is now thought unsound to apply a correction to the general mean, if observations on some zeros are more consistent than those on others, as this will give undue weight to the graduation error of the apparently better zeros. If observations on any zero are exceptionally inconsistent, the weight of that zero is improved by repeat observations (see para 58).

GRADUATION ERROR
of
12-inch Theodolite No. II

Fig. 26

f	$\frac{\Sigma e}{2\Sigma b}$	Running Total =Graduation Error	At Circle Reading
0°·0	+0·03	+0″·03	3°3/4
7·5	+0·09	+0·12	11 1/4
15·0	+0·01	+0·13	18 3/4
22·5	+0·08	+0·21	26 1/4
30·0	+0·16	+0·37	33 3/4
37·5	+0·04	+0·41	41 1/4
45·0	+0·08	+0·49	48 3/4
25·5	+0·08	+0·57	56 1/4
60·0	−0·02	+0·55	63 3/4
67·5	−0·19	+0·36	71 1/4
75·0	−0·03	+0·33	78 3/4
82·5	−0·08	+0·25	86 1/4
90·0	+0·06	+0·31	93 3/4
97·5	+0·30	+0·61	101 1/4
105·0	+0·15	+0·76	108 3/4
112·5	+0·22	+0·98	116 1/4
120·0	+0·16	+1·14	123 3/4
127·5	−0·01	+1·13	131 1/4
135·0	−0·18	+0·95	138 3/4
142·5	−0·27	+0·68	146 1/4
150·0	−0·24	+0·44	153 3/4
157·5	−0·25	+0·19	161 1/4
165·0	−0·09	+0·10	168 3/4
172·5	−0·10	+0·00	176 1/4
180·0	+0·03	+0·03	3 3/4



REG. No. 426. M.D. 31.

H.S.I.O. Dehra Dūn.

Fig. 27

7A Trian. **Survey of India** Page

No. 15 PARTY (Triangulation) SEASON 1929-30

Computation of Weights of Observed Angles.

At Sintaung H. Station
Angle between Myinmoti H. Station
and Loi Phui H. Station
= 40° 08′ 02″·27

Circle Reading of A micrometer: Left Arm (1)	Circle Reading of A micrometer: Right Arm (2)	Z.M. – G.M. from Abstract (3)	Graduation error*: Left Arm (4)	Graduation error*: Right Arm (5)	(3)+(4)–(5) = v (6)
360°	40°	+0″·8	0″·0	+0″·7	+0″·1
15	55	+1·3	+0·2	+1·0	+0·5
30	70	+2·2	+0·4	+0·9	+1·7
45	85	−1·1	+0·8	+0·5	−0·8
60	100	−2·2	+1·1	+0·9	−2·0
75	115	+1·4	+0·7	+1·6	+0·5
90	130	+1·5	+0·5	+2·3	−0·3
105	145	+0·1	+1·2	+1·7	−0·4
120	160	−2·3	+1·9	+0·7	−1·1
135	175	−1·7	+2·2	+0·2	+0·3
150	190	+0·1	+1·3	+0·2	+1·2
165	205	−0·2	+0·4	+0·2	0·0

Sum (without regard to sign) = Σ|v| =	8.9
F_2† =	0.021
Probable error = Σ|v| × F_2 =	0.187
[Probable error]² =	0.035
Reciprocal Weight = u = 2·20 × [p.e.]² =	0.077
Weight =	13·0

At Station
Angle between Station
and Station
= ° ′ ″

Circle Reading of A micrometer: Left Arm (1)	Circle Reading of A micrometer: Right Arm (2)	Z.M. – G.M. from Abstract (3)	Graduation error*: Left Arm (4)	Graduation error*: Right Arm (5)	(3)+(4)–(5) = v (6)

Sum (without regard to sign) = Σ|v| =
F_2† =
Probable error = Σ|v| × F_2 =
[Probable error]² =
Reciprocal Weight = u = 2·20 × [p.e.]² =
Weight =

At Station
Angle between Station
and Station
= ° ′ ″

Circle Reading of A micrometer: Left Arm (1)	Circle Reading of A micrometer: Right Arm (2)	Z.M. – G.M. from Abstract (3)	Graduation error*: Left Arm (4)	Graduation error*: Right Arm (5)	(3)+(4)–(5) = v (6)

Sum (without regard to sign) = Σ|v| =
F_2† =
Probable error = Σ|v| × F_2 =
[Probable error]² =
Reciprocal Weight = u = 2·20 × [p.e.]² =
Weight =

* From running total in the analysis of graduation error, or from Chart. † From 15A Math., Auxiliary Tables, 5th Edition, Part II: n = number of entries in (6).

NOTE:—Nearest degree in (1) and (2) One decimal of a second in (3) to (6).

Computed by R.K.B. *Date* 20.12.30 *Checked by* S.I.A.

line by line, to form a column 6 for the double angle, from which the weight can be deduced in the usual way.

The reciprocal weight u is also recorded.

Spherical Excess.—The spherical excess is next computed on form 8 Trian. The triangles should be computed in order from one end of the series to the other. Each deduction will then furnish the log side which is necessary for the computation of the next. On this form angles are recorded to the nearest second only, and 4-figure logarithms are used. The spherical excess is required to two decimals of a second.

Grinding quadrilaterals.—The next step is to make the figural adjustments. A braced quadrilateral is ground on form 9 Trian. (see fig. 28). At the top of the form is a diagrammatic quadrilateral, against the corners of which are entered the names of the stations of the figure concerned. In the top right hand corner are entered the observed* angles (from the abstract) with their reciprocal weights.

The triangular errors are then computed at the left hand side of the form, and the side closure† at the right and centre. The columns headed "Final Error" and "Test" are of course left blank at this stage. In the columns s are entered the changes in the 7th figure of each log sine, corresponding to a change of 1 second in the angle. The reciprocal weights at the head of Table I are next filled in, and also the entries -- s_1, etc., in row D.

Tables II and III are next completed. It must be noted that in conformity with the general custom in all Survey of India forms, a + sign is definitely *plus*, but a – sign may be left unchanged or converted to + as may be appropriate.

The entries in the top four lines of Table IV are the horizontal sums of the rows of Table III, these four lines being the coefficients of f_1, f_2, f_3 and f_4, the unknowns in the normal equations. The sign = may be supposed to precede the column e, in each line. In this last column are entered e_1, e_2, e_3 and e_4, the triangular and side errors.

The four normal equations are then solved automatically by the usual method as explained in text books. The rule is as follows:—Multiply the first equation through by the coefficient of f_2, divide by the coefficient of f_1, and record it with the sign changed in the first line below, omitting the resulting coefficient of f_1. The next line would be obtained by multiplying the first equation through by the coefficient of f_3 and dividing by the coefficient of f_1; but as the coefficient of f_3 is 0, the resulting quantities will be 0, and they

* The word "concluded angle" which used to appear at the top left hand corner of the form is now synonymous with observed angle. See footnote to page 80. These observed angles should be corrected for deflection, height, and geodesic, if possible.

† It is not necessary to correct the observed angles for spherical excess before taking log sines to find the side closure.

have been so printed in the form. The third line is obtained by multiplying the first equation through by the coefficient of f_4, dividing by that of f_1, and changing sign. The coefficients need not be entered where no provision is made for them. Add the coefficients in the first line of the new group to those of equation 2 and record below: then add the coefficients in the second line of the new group to those of equation 3 and record, and treat the fourth equation similarly. We have now three equations involving three unknown quantities, f_2, f_3 and f_4. Going through a similar process to that already described, we come to two equations involving two unknown quantities f_3 and f_4, and lastly we reduce the equations to one involving one unknown quantity f_4. Dividing out by its coefficient we have the value of f_4. This brings us to the bottom of the form. The remainder of Table IV is found above and to the right. Taking the last line but one, fill it up from the first equation in the group involving two unknown quantities, f_3 and f_4, transferring the term involving f_4 over to the other side. Then substitute for f_4, add the two terms, and divide by the coefficient of f_3; this gives f_3. To find f_2, employ the first of the group of equations involving the three unknown quantities f_2, f_3 and f_4, and enter the third line of the last portion of Table IV: and similarly for f_1. Then enter Table V and multiply the quantities in Table I by the values of f_1, f_2, etc., as indicated. The resulting values obtained from this table are the most probable errors of the angles.

The errors obtained are then entered in the test column left blank in the earlier part of the form. In the case of the triangular conditions the sum of the three errors should equal the triangular error. In the case of the side conditions the errors are each multiplied by the appropriate s, and the left and right hand parts of the table are summed to give S_l' and S_r' respectively. Then $S_r' - S_l'$ should equal e_4. These tests must prove almost exactly unless mistakes have been made.

The number of decimal places contained in the different stages of the computation should be in accordance with the example given in fig. 28. This example is appropriate to the magnitudes of the reciprocal weights generally found in primary triangulation, namely between zero and 2·0, with some values in each figure at least greater than 0·1. If the form is used for triangulation of unusual quality it may be convenient to multiply or divide all the values of u in any one figure by some constant (e.g. 10) before entering them in the form: it is clear that this is a legitimate procedure, in no way affecting the distribution of error, but the fact that such action has been taken should be recorded at the top of the form, to avert subsequent misunderstanding.

9 Trian.

Survey of India

Fig. 28

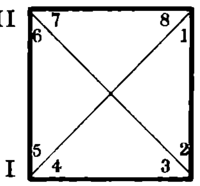
No. 15 PARTY (Triangulation) SEASON 1929 - 30.

Name of Series Mong Hsat

Reduction of Myinmati H.S. – Lethataung H.S. *Quadrilateral by the Method of Minimum Squares.*

Explanation of Symbols.	
X_n denotes General Mean from Abstract.	ε denotes Spherical Excess.
x_n " most Probable Error of X_n.	e " Error.
u_n " Reciprocal Weight "	f " Factor (indeterminate).
δ_n " change of log sin X_n for 1″.	
$\delta_{m,n}$ " " " $\{X_m + X_n\}$ for 1″.	

Myinmati H.S. or III — or IV Loi Phui H.S.



Sintaung H.S. or I — or II Lethataung H.S.

Statement of Data.

No.	X	u	No.	X	u
	° ′ ″			° ′ ″	
1	36 35 14.16	0.29	5	40 08 02.27	0.08
2	46 08 58.93	0.32	6	42 36 11.86	0.13
3	59 41 25.01	0.11	7	45 07 14.23	0.20
4	37 34 25.87	0.14	8	52 08 38.09	0.19

Triangular Error = Sum of Angles − (180° + ε).

Seconds of X Δ I. II. IV	Final Errors tested	Seconds of X Δ I. II. III	Final Errors tested	Seconds of X Δ I. III. IV	Final Errors tested
X_1 14.16	−0.30	X_3 25.01	−0.29	X_5 2.27	−0.12
X_2+X_3 23.94	+0.07	X_4+X_5 28.14	−0.25	X_6+X_7 26.09	+0.25
X_4 25.87	−0.13	X_6 11.86	+0.06	X_8 38.09	+0.48
Sum 3.97		Sum 5.01		Sum 6.45	
ε 4.33		ε 5.49		ε 5.84	
e_1 = − 0.36	− 0.36	e_2 = − 0.48	− 0.48	e_3 = + 0.61	+ 0.61

Side Error = 10,000,000 × (Sum of log sines on right − Sum of log sines on left).

X	log sine	δ	Test: Final Error	Test: Change in log sine	X	log sine	δ	Test: Final Error	Test: Change in log sine
X_1 36 35 14.16	$\bar{1}$.775 2801	+29	−0.30	−8.7	X_2+X_3 105 50 23.94	$\bar{1}$.983 1876	−6	+0.07	−0.4
X_3 59 41 25.01	$\bar{1}$.936 1667	+13	−0.29	−3.8	X_6 42 36 11.86	$\bar{1}$.830 5363	+23	+0.06	+1.4
X_6+X_7 87 43 26.09	$\bar{1}$.999 6572	+1	+0.25	+0.3	X_8 52 08 38.09	$\bar{1}$.897 3822	+16	+0.48	+7.7
Sum = S_l = $\bar{1}$.711 1040			S'_l = −12.2		Sum = S_r = $\bar{1}$.711 1061			S'_r = +8.7	
e_4 = 10,000,000 × {$S_r - S_l$} = + 21					$S'_r - S'_l$ = + 21				

	x_1 u_1=0.29	x_2 u_2=0.32	x_3 u_3=0.11	x_4 u_4=0.14	x_5 u_5=0.08	x_6 u_6=0.13	x_7 u_7=0.20	x_8 u_8=0.19
Table I.—Numerical Statement of Coefficients in Equations of Condition.								
A	+1	+1	+1	+1				
B	...	...	+1	+1	+1	+1		
C	...	...	...	...	+1	+1	+1	+1
D	−29	−6	−19	...	...	+22	−1	+16
	$-\delta_1$	$+\delta_{2,3}$	$+\delta_{2,3}-\delta_3$			$-\delta_{6,7}+\delta_6$	$-\delta_{6,7}$	$+\delta_8$
Table II.—Products of Coefficients in Table I and the corresponding u.								
A.u	+0.29	+0.32	+0.11	+0.14				
B.u	...	...	+0.11	+0.14	+0.08	+0.13		
C.u	...	...	...	...	+0.08	+0.13	+0.20	+0.19
D.u	−8.41	−1.92	−2.09	...	...	+2.86	−0.20	+3.04
Table III.—Products of quantities in Table II and those in corresponding cols. of Table I.								
Au.A	+0.29	+0.32	+0.11	+0.14				
Au.B	...	...	+0.11	+0.14				
Au.D	−8.41	−1.92	−2.09					
Bu.B	...	...	+0.11	+0.14	+0.08	+0.13		
Bu.C	...	...	...	...	+0.08	+0.13		
Bu.D	...	...	−2.09	...	...	+2.86		
Cu.C	...	...	...	...	+0.08	+0.13	+0.20	+0.19
Cu.D	...	...	...	...	...	+2.86	−0.20	+3.04
Du.D	+243.89	+11.52	+39.71	...	...	+62.92	+0.20	+48.64

Table IV.—Equations between Factors.

Eqn. No.	Coefficient of f_1	f_2	f_3	f_4	e
1	+0.86	+0.25	...	−12.42	−0.36
2	...	+0.46	+0.21	+0.77	−0.48
3	...	*	+0.60	+5.70	+0.61
4	...			+406.88	+21
*The quantities below the diagonal are not to be recorded, being identical with those above.		−0.0727	...	+3.6105	+0.10465
			0.0000	0 0000 0	0 00000
				−179.3679	−5.19907
		+0.3873	+0.21	+4.3805	−0.37535
			+0.60	+5.70	+0.61
				+227.5121	+15.80093
			−0.1139	−2.3752	+0.20352
				−49.5450	+4.24534
			+0.4861	+3.3248	+0.81352
				+177.9671	+20.04627
				−22.7408	−5.56427
				+155.2263	+14.48200
				f_4 = + 0.09330	

Horl. Sum		Eqn.
+ 0.86	= Coeff. of f_1 in	1
+ 0.25	= " " f_2 "	1
− 12.42	= " " f_4 "	1
+ 0.46	= " " f_2 "	2
+ 0.21	= " " f_3 "	2
+ 0.77	= " " f_4 "	2
+ 0.60	= " " f_3 "	3
+ 5.70	= " " f_4 "	3
+ 406.88	= " " f_4 "	4

$+0.86\,f_1 = -0.25\,f_2 + 12.42\,f_4 - 0.36$

$= +0.64644 + 1.15874 - 0.36 = +1.44518\;;\; f_1 = +1.68044$

$+0.3873\,f_2 = -0.21\,f_3 - 4.3805\,f_4 - 0.37535$

$= -0.21744 - 0.40868 - 0.37535 = -1.00147\;;\; f_2 = -2.58577$

$+0.4861\,f_3 = -3.3248\,f_4 + 0.81352$

$= -0.31019 + 0.81352 = +0.50333 \qquad f_3 = +1.0354$

Table V.—Final Determination of Errors †

	x_1 u_1 = 0.29	x_2 u_2 = 0.32	x_3 u_3 = 0.11	x_4 u_4 = 0.14
f_1.A	+ 1.6804	+ 1.6804	+ 1.6804	+ 1.6804
f_2.B	...	...	− 2.5858	− 2.5858
f_3.C	...	...	...	...
f_4.D	− 2.7056	− 0.5598	− 1.7726	...
Sum	− 1.0251	+ 1.1207	− 2.6780	− 0.9053
Sum × u = x″	− 0.297	+ 0.359	− 0.295	− 0.127

	x_5 u_5 = 0.08	x_6 u_6 = 0.13	x_7 u_7 = 0.20	x_8 u_8 = 0.19
f_1.A	...	...	...	...
f_2.B	− 2.5858	− 2.5858	...	...
f_3.C	+ 1.0355	+ 1.0355	+ 1.0355	+ 1.0355
f_4.D	...	+ 2.0525	− 0.0933	+ 1.4927
Sum	− 1.5503	+ 0.5022	+ 0.9422	+ 2.5282
Sum × u = x″	− 0.124	+ 0.065	+ 0.188	+ 0.480

† Note.—The Errors obtained in this table (Table V) are to be applied with *changed* signs, as corrections, in the Triangle sheets (11 Trian.).

Computed and Compared by R.K.B. and S.I.A. 17. 12. 1930

Grinding polygons.—A hexagon is ground on form 10 Trian. A pentagon or centred quadrilateral is ground on the same form, not all the columns being required. A similar form can easily be improvised for a heptagon or any other simple centred figure without diagonals; but for more complicated figures a special form must be improvised (see para 74).

The method of computation on form 10 Trian. will be clear from the explanation of 9 Trian. given above. In case of doubt examples can always be found among the records in the Computing Office.

Computation of log sides.—Form 11 Trian. (solution of triangles) is next set up, the angles being taken from the abstract form and not from the grinding form: this is in order to provide an independent check on the copying of the angles. Where the series consists of simple triangles, the order in which the triangles should be set up is immediately obvious. Where the series consists of quadrilaterals or other figures, it is not imperative to set up more triangles than are sufficient to carry forward the log side and to fix all the stations, since after adjustment has been carried out different pairs of triangles will essentially give the same result. Nevertheless all possible triangles should always be set up, in order that the agreement of the values of common sides, derived by different routes, may independently check the accuracy of the computation. Discrepancies of as much as 3 in the last (7th) decimal may occur: in such cases the mean should be accepted.

One third of the spherical excess has to be deducted from each angle of a triangle, and the corrections derived from the grinding forms have also to be applied. In the case of a simple triangle the corrections are derived by dividing the triangular error e among the angles in proportion to the reciprocal weights. If the latter are u_1, u_2 and u_3 the corrections are:—

$$\frac{u_1\, e}{u_1+u_2+u_3}, \quad \frac{u_2\, e}{u_1+u_2+u_3} \text{ and } \frac{u_3\, e}{u_1+u_2+u_3}.$$

In this form the three stations of a triangle should always be set up in clockwise order, the station opposite the known side appearing last (see fig. 29).

Latitudes and longitudes.—The latitude and longitude of each station should be computed by two deductions on form 13A Trian. (see fig. 30). Two deductions are sufficient, however many rays are available, since when figures have been adjusted, all rays will necessarily give identically the same result. The two rays employed should form two sides of a triangle ABC, C being the unfixed point, and A being that which precedes B in clockwise order. Then to deduce the azimuth of C at A and B, it is necessary to add the spherical angle BAC to the known azimuth of B at A, and to subtract the angle ABC from the azimuth of A at B.

To avoid confusion the computer should have a chart of the triangulation in front of him when deducing these azimuths, although strict adherence to the rule of setting up forms 11 and 13A Trian. makes the rule of signs almost automatic.

The number of decimals to be used in the computation is indicated in the example given in fig. 30. The form is self-explanatory and no special instructions are required. The small corrections computed in lines 60-65 and 67-72 may be obtained from nomograms. (Tables 7 & 8 Geod. of Auxiliary Tables, Part IV).

Heights.—The computation of heights is carried out on form 16 Trian. (see fig. 31). Vertical angles will ordinarily have been fully observed at both ends of every ray, and the height differences should be deduced from all observed vertical angles (see para 77). When taking the mean of different determinations of the height of a station, most weight should ordinarily be given to the shortest ray: other things being equal the weight is inversely proportional to the square of the length of the ray. If the observed vertical angles at any station have shown exceptional variation from day to day, or if the heights determined from the two ends of a ray are exceptionally divergent, the determination must be given correspondingly low weight. Observations not made at the hour of minimum refraction will generally be rejected.

Provided the observed vertical angles from one station to another are reasonably constant from day to day, the computation should be carried out with the mean of all the observed vertical angles, and the mean temperature and pressure. But if observations are divergent, it will be best to compute each day separately, so that the rejection of any one day can be considered.

Heights of instrument and signal should invariably be above the upper markstone of the stations, i.e. the markstone at the top of the 40-inch circular isolated pillar, or corresponding structure. This must not be confused with the smaller, square "protecting pillar" which has sometimes been built (see pages 8, 9 and 10 of the 1902 edition of this handbook). All published heights refer to the upper markstone.

Synopsis.—The synopsis of the latitude, longitude and height of each station, with the distances and azimuths of adjacent stations is prepared on form 21 Trian. When junction is made with any previously observed geodetic triangulation, the previously accepted values of these quantities should be entered in red ink below the new values, and a note should be made of the source from which they have been obtained.

Fig. 29

11 Trian.

Survey of India

No. 15 **PARTY** (Triangulation) Mong Hsat Series **SEASON 1929-30**

Computation of Principal Triangles.

Reciprocal Weight	Station		Angle Observed	Spherical Excess	Correction	Seconds of Angle for Computation	Log Sine*	Distance in Log Feet	Distance in Miles
No. 1							5 134 7181		
.	Lethataung	H.S.	59 41 25 01	-1 83	+0 29	23 47	$\bar{1}$ 936 1649	5 240 3511	32.9
.	Sintaung	H.S.	77 42 28 14	-1 83	+0 25	26 56	$\bar{1}$ 989 9270	5 294 1132	37.3
.	Myinmati	H.S.	42 36 11 86	-1 83	-0 06	09 97	0 169 4681	Feet	
			180 00 05 01	-5 49	+0 48	00·00			
No. 2							5 240 3511		
.	Sintaung	H.S.	40 08 02 27	-1 94	+0 12	00 45	$\bar{1}$ 809 2702	5 152 2431	26.9
.	Myinmati	H.S.	87 43 26 09	-1 95	-0 25	23 89	$\bar{1}$ 999 6571	5 342 6300	41.7
.	Loi Phui	H.S.	52 08 38 09	-1 95	-0 48	35 66	0 102 6218	Feet	
			180 00 06 45	-5 84	-0 61	00·00			
No. 3							5 134 7181		
.	Lethataung	H.S.	105 50 23 94	-1 45	-0 07	22 42	$\bar{1}$ 983 1885	5 342 6297	41.7
.	Sintaung	H.S.	37 34 25 87	-1 44	+0 13	24 56	$\bar{1}$ 785 1721	5 144 6133	26.4
.	Loi Phui	H.S.	36 35 14 16	-1 44	+0 30	13 02	0 224 7231	Feet	
			180 00 03 97	-4 33	+0 36	00·00			
No. 4							5 294 1132		
.	Lethataung	H.S.	46 08 58 93	-1 56	-0 36	57 01	$\bar{1}$ 858 0231	5 152 2429	26.9
.	Myinmati	H.S.	45 07 14 23	-1 56	-0 19	12 48	$\bar{1}$ 850 3937	5 144 6135	26.4
.	Loi Phui	H.S.	88 43 52 25	-1 57	-0 17	50 51	0 000 1066	Feet	
			180 00 05 42	-4 69	-0 72	00 00			
No. 5							5 152 2430		
0.14	Loi Phui	H.S.	60 03 43 33	-1 35	+0 02	42 00	$\bar{1}$ 937 8002	5 147 7275	26·6
0.10	Myinmati	H.S.	58 49 12 90	-1 34	+0 01	11 57	$\bar{1}$ 932 2423	5 142 1696	26·3
0.08	Loi Maw	H.S.	61 07 07 77	-1 35	+0 01	06 43	0 057 6843	Feet	
			180 00 04 00	-4 04	+0 04	00·00			
No.									
.									
.									
.								Feet	
No.									
.									
.									
.								Feet	

NOTE:—The names of Hill Stations should be followed by H. S., of Tower Stations by T. S., and of Platform Stations by P. S.
* In every third line in this column enter the Log. cosecant in place of the Log sine of the Angle.

Computed and compared by R. K. B. and S. I. A. 18.12 1930.

Fig. 30

13A Trian. | **Survey of India** | **Page**

NO. 15 PARTY (Triangulation) SEASON 1929-30

Computation of Latitudes, Longitudes and Azimuths of Geodetic Stations.

No.	Reference	A			B		
1	Reference ‖ Nos.	Δ No. 4			Del. No. 2		
2	Station A	Lethataung H.S.					
3	" B	Myinmati H.S.					
4	" C	Loi Phui H.S.					
5	Azimuth at A of B ‖ B of A	217°	12′	36.03	37°	19′	52.66
6	∠ BAC ‖ ∠ ABC (1)	+46	08	57.01	−45	07	12.48
7	Spherical Excess = $\frac{\epsilon}{3}$ (2)	+ 0	00	01.56	− 0	00	01.56
8	Sum = **A** = Az. of C at A ‖ at B	263	21	34.60	352	12	38.62
9	Latitude of A = λ_A ‖ B = λ_B	20	10	28.333	20	36	21.004
10	Longitude of A = L_A ‖ B = L_B	96	40	37.032	97	01	30.212
11	log AC ‖ log BC	5	144	6134	5	152	2430
12	log ν for λ_A ‖ log ν for λ_B (3)	7	320	7941	7	320	8011
13	(11) − (12) = log σ	$\bar{3}$	823	8193	$\bar{3}$	831	4419
14	log for sin σ (4)	$\bar{8}$	685	5716	$\bar{8}$	685	5715
15	log 50 cosec 1″	7	013	3951	7	013	3951
16	Sum = log 50 sin σ	$\bar{1}$	522	7860	$\bar{1}$	530	4085
17	log cos **A**	$\bar{1}$	063	0982	$\bar{1}$	995	9743
18	Sum = log 50 X	$\bar{2}$	585	8842	$\bar{1}$	526	3828
19	log tan λ (5 fig)	$\bar{1}$	565	17	$\bar{1}$	575	18
20	log 25 (5 fig)	1	397	94	1	397	94
21	2 log σ (log σ from 13) (5 fig)	$\bar{5}$	647	64	$\bar{5}$	662	88
22	Sum = log 25 Y (5 fig)	$\bar{4}$	610	75	$\bar{4}$	636	00
23	50 X (anti log line 18) (5) (7 decimals)	+0	·038	5376	−0	·336	0336
24	25 Y (anti log line 22) (7 decimals)	+0	·000	4081	+0	·000	4325
25	50 X − 25 Y = 100 P (7 decimals)	+0	·038	1295	−0	·336	4661
26	log P = log 100P − log 100 (5-fig.)	$\bar{4}$	581	2611	$\bar{3}$	526	9412
27	log tan λ (line 19) (5-fig.)	$\bar{1}$	565	17	$\bar{1}$	575	18
28	Sum = log Pt (5-fig.)	$\bar{4}$	146	43	$\bar{3}$	102	12
29	Pt. (6) (7 decimals)	+0	·000	1401	−0	·001	2651
30	$\left(\frac{100P}{100}\right)^2$ (7) (7 decimals)	0	·000	0001	0	·000	0113
31	Algebraic Sum = v (7 decimals)	+0	·000	1402	−0	·001	2538
32	3v	+0	·000	4206	−0	·003	7614
33	1 − 3v	0	·999	5794	1	·003	7614
34	log (1 − 3v)	$\bar{1}$	999	8174	0	001	6305
35	½ log (1 − 3v)	$\bar{1}$	999	9391	0	000	5435
36	(26) − (35) = log tan $\frac{\Delta_1\lambda}{2}$	$\bar{4}$	581	3220	$\bar{3}$	526	3977
37	log for $\left(\tan\frac{\Delta_1\lambda}{2}\right)$ (4)	$\bar{8}$	685	5749	$\bar{8}$	685	5765
38	(36) − (37) = log $\frac{\Delta_1\lambda''}{2}$	1	895	7471	2	840	8212
39	½ $\Delta_1\lambda''$ (3 decimals) (6)	+	78·6588		−	693·1403	
40	$\Delta_1\lambda''$ (6)	+	157·3176		−	1386·2806	

No.	Reference	A			B		
41	$\Delta_1\lambda$ (6)	+ 0°	02′	37″.318	− 0°	23′	06″.281
42	(9) + (41) = λ + $\Delta_1\lambda$	20	13	05.651	20	13	14.723
43	log sec (λ + $\Delta_1\lambda$)	0	027	6198	0	027	6269
44	log sin **A**	$\bar{1}$	997	0767	$\bar{1}$	132	0357
45	log 50 sin σ (line 16)	$\bar{1}$	522	7860	$\bar{1}$	530	4085
46	co-log 50	$\bar{2}$	301	0300	$\bar{2}$	301	0300
47	Sum = log sin $\Delta_1 L$	$\bar{3}$	848	5125	$\bar{4}$	991	1011
48	log for (sin $\Delta_1 L$) (4)	$\bar{8}$	685	5712	$\bar{8}$	685	5748
49	(47) − (48) = log $\Delta_1 L''$	3	162	9413	2	305	5263
50	log sec $\frac{\Delta_1\lambda}{2}$	0	000	0000	0	000	0025
51	log sin$\left(\lambda + \frac{\Delta_1\lambda}{2}\right)$	$\bar{1}$	538	1199	$\bar{1}$	542	5634
52	Sum = log Δ**A**.	2	701	0612	1	848	0922
53	Δ**A**. (to nearest 10″)	500″			70″		
54	$\Delta_1 L$ (anti log line 49) 3 dec. (8)	+ 1455·262			+ 202·081		
55	log for $\left(\tan\frac{\Delta_1 L}{2}\right)$ (4)	$\bar{8}$	685	5766	$\bar{8}$	685	5749
56	log for $\left(\tan\frac{\Delta A.}{2}\right)$ (4)	$\bar{8}$	685	5751	$\bar{8}$	685	5749
57	(55) − (56)	0	000	0015	0	000	0000
58	(52) + (57) = log Δ_1**A**	2	701	0627	1	848	0922
59	Δ_1**A** (8) 2 dec.	+ 502″·42			+ 70″·48		
60	log constant (9) 4-fig.	2	537	3	2	537	3
61	2 log cos λ (9) 4-fig.	$\bar{1}$	945	0	$\bar{1}$	942	6
62	log sin 2**A** (9) 4-fig.	$\bar{1}$	361	2	$\bar{1}$	429	0
63	2 log σ (line 21) (9) 4-fig.	$\bar{5}$	647	6	$\bar{5}$	662	9
64	Sum = log δ**A** (9) 4-fig.	$\bar{3}$	491	1	$\bar{3}$	571	8
65	δ**A** (10)	+ 0″·00			− 0″·00		
66	Δ_1**A** + δ**A** = (59) + (65)	+ 502·42			+ 70·48		
67	log constant (4-fig.)	3	014	4	3	014	4
68	log sin 2λ (4-fig.)	$\bar{1}$	811	2	$\bar{1}$	818	8
69	2 log σ (line 21) (4-fig.)	$\bar{5}$	647	6	$\bar{5}$	662	9
70	2 log cos **A** (4-fig.)	$\bar{2}$	126	2	$\bar{1}$	991	9
71	Sum = log δλ (4-fig.)	$\bar{4}$	599	4	$\bar{2}$	488	0
72	δλ	+ 0″·0004			+ 0″·0308		
73	$\Delta\lambda_0$ = (40) − (72)	+ 157·317			− 1386·311		
74	log $\Delta\lambda_0$	2	196	7756	3	141	8606
75	log $\frac{\nu}{\rho}$ (11)	0	002	5494	0	002	5351
76	Sum = log Δλ″	2	199	3250	3	144	3957
77	Δλ″ (12)	+ 158″·243			− 1394·427		
78	λ_C = Lat. of C λ_A + (77) ‖ λ_B + (77)	20°	13′	06″.576	20°	13′	06″.577
79	L_C = Long. of C L_A + (54) ‖ L_B + (54)	97	04	52.294	97	04	52.293
80	180° + (8) + (66) = Az. at C	83	29	57.02	172	13	49.10

(1) Angle used for computation in 11 Trian. should be entered; for sign see chart. (2) Same sign as the angles in line 6. (3) ν = Normal, from 1 Geod., Aux Tables, Part IV, 5th Edn. (4) From 5 Geod., Aux. Tables, Part IV. 5th Edn. (5) This is + if **A** is between 90° and 270°, otherwise −. (6) Same sign as 100 P in line 25. (7) From 10 Math. Aux. Tables, Part II, 5th Edn. (8) This is + if **A** is between 180° and 360°, otherwise −. (9) Charts 7 and 8 Geod. for δ**A** and δλ respectively may be used as an alternative to computation of lines 60 to 65 and 67 to 72. (10) This is + if **A** is between 0° and 90° or 180° and 270°, otherwise −. (11) From 6 Geod., Aux. Tables, Part IV, 5th Edn. (12) Same sign as $\Delta\lambda_0$ in line 73.

Computed by R. K. B. *Date* 19. 1. 31. *Checked by* S. I. A.

16 Trian. **Survey of India** Page

15 PARTY (Triangulation) SEASON 1929-30.

Series Mong Hsat *by* Mr. R. B. Mathur B.A.

Height and Refraction computations of Geodetic *Triangulation.*

Stations or Points.

Deduction Nos.*	9 A	10 B			
Fixed station A.	Myinmati H.S.	Myinmati H.S.			
Deduced station B.	Loi Phui H.S.	Loi Phui H.S.			
Baro. reduced from 18 Sur.	23·62 inches	23·96 inches	· inches	· inches	inches
Thermometer	63°·6	70°·1	°	°	°
Hour and date of obsn.	13h 30d 12m 1929	14h 24d 12m 1929	h d m 19	h d m 19	h d m 19
Refn. coefft. (assumed)(1) = k	·	·	·	·	·
Azimuth of B = A	352°	352°	°	°	°
4 fig. logs: log (1 − 2k)(1)	$\bar{1}$ 9366	$\bar{1}$ 9376			
log from 6 Sur. (2)	$\bar{3}$ 6951	3 6951	$\bar{3}$ 69	$\bar{3}$ 69	$\bar{3}$ 69
log side	5 1522	5 1522			
Sum = log ψ seconds	2 7839	2 7849			
ψ = ½ χ (1 − 2k)	+0° 10′ 08″ 0	+0° 10′ 09″ 4	+ ° ′ ″	+ ° ′ ″	+ ° ′ ″
Observed altitude = a_0 (3)	−0 18 38 1	−0 02 07 4	−	−	−
$a = a_0 + \psi$	−0 08 30 1	+0 08 02 0	−	−	−
a' = a in minutes (4)	8·500	8·033	·	·	·
log a'	0 929 42	0 904 88			
log for a from 2 Math(5) (tangents)	$\bar{4}$ 463 73	$\bar{4}$ 463 73	$\bar{4}$ 46	$\bar{4}$ 46	$\bar{4}$ 46
log side	5 152 24	5 152 24			
Sum = log Δh	2 545 39	2 520 85			
Δh(6)	− 351·1	+ 331·6	−	−	−
δh from 7 Sur (7)	− 0·1	+ 0·1	−	−	−
i = Height of instrument (8)	+ 5·2	+ 5·2	+	+	+
g = Height of signal (9)	− 2·6	− 2·5	−	−	−
Sum = $h_b - H_a$ (10)	− 348·6	+ 334·6	−	−	−
Height of A = H_a	6460	6460			
Sum = Ht. of B = h_b	6111	6125			
Mean Height of B = H_b	6118				
$H_b - h_b = \delta h_b$ (10)	−	−	−	−	−
4 fig. logs: log δh_b					
log (0·4343 cosec 1″)	4 9522	4 9522	4 9522	4 9522	4 9522
Sum = (a)					
log ψ seconds + log side = (b)					
Difference (a) − (b) = log x					
x = corrn. for log (1−2k) (11)	−	−	−	−	−
log (1 − 2k)					
Sum = log (1 − 2k₁) (12)					
1 − 2k₁ (12)	·	·	·	·	·
k₁ = refraction coefficient (12)	·	·	·	·	·

* Enter "A" or "B" after deduction number to indicate the station at which observations were made.
(1) Table 5 Sur., Aux. Tables, 5th Edn., Part III. Tabular values may be used if k is not otherwise known.
(2) Aux. Tables, 5th Edn., Part III. Enter table with arguments A and λ (values nearest degree).
(3) a_0 is + or − according as the observed angle is *elevation* or *depression*.
(4) Aux. Tables, 5th Edn., Part II: *vide* page (11) for conversion of seconds into minutes.
(5) Aux. Tables, 5th Edn., Part II: the sum of this and log a' = log tan a.
(6) Δh has same sign as a.
(7) Aux. Tables, 5th Edn., Part III: δh has same sign as Δh.
(8) Sign for instrument, +
(9) Sign for signal, −.
(10) Change sign of Sum and δh_b if B is station of observation.
(11) This has same sign as δh_b.
(12) log $(1-2k_1)$ & $1-2k_1$ are the quantities which correspond in log and number to *the mean height* accepted: and k_1 is the refraction coefficient which reproduces that height.

Computed and compared by R.K.B. and S.I.A. 1. 4. *1931*

At this stage of the computation no adjustment to old work is carried out. After adjustment (in the Computing Office or Triangulation Party, as the case may be) a fresh synopsis will be prepared.

79. Intersected points.—Intersected points should be computed in the same way as stations. There is of course no weighting, grinding, or distribution of triangular error. The third angle of the triangle is deduced from the two observed angles, spherical excess being taken into account. The log sides are computed on form 11 Trian., and latitude and longitude on form 13A Trian.* The heights are computed on form 8 Topo., and the synopsis is made on form 9 Topo.

The positions of intersected points must be corrected for any subsequent adjustment of the series. If the adjustment is being carried out in the triangulation party, it will be best to postpone computation of latitude and longitude of intersected points until the adjustment is complete.

Form 11 Trian. should be set up for as many independent triangles as are available in order to obtain independent values of the common sides, as a check on the accuracy of observation. The observer should decide what values or combination of values of the common sides are to be accepted, giving due consideration to the conditions under which the point was observed. Form 13A Trian. will be computed with two rays only, the most reliable. The acceptance of mean values for the lengths of these rays may result in imperfect agreement between the two deductions on this form, but the distance between the two positions deduced must be less than the difference between the different values obtained for the sides which have been meaned together.

80. Spelling of names.—The correct spelling of the names of all new stations should be verified by reference to the District Officers or Political Agents. A list should be prepared and attached to the computations, showing the names provisionally adopted in the angle books and computations, and those finally adopted. The names of old G.T. stations can never be changed, although better names can be added in brackets after the old names.

81. History Sheet.—A tabular statement should be prepared giving the following details:—

(*a*) Name and number of series.

(*b*) Season of observation.

(*c*) Name of observer.

(*d*) Name of officer by whom computations have been scrutinised (see para 88).

* The observed angles may be entered directly in line 6, line 7 being left blank.

(*e*) Name of Officer in charge of the party.
(*f*) Name of the Director of the Geodetic Branch.
(*g*) The instrument used.
(*h*) The opening sides.
(*i*) The closing sides.
(*j*) The number of stations observed at.
(*k*) ,, new stations.
(*l*) The length of the series in miles.
(*m*) The number of square miles covered by the series.
(*n*) ,, each type of figure.
(*o*) The average length of side in miles.
(*p*) ,, triangular error.
(*q*) The maximum triangular error.
(*r*) The values of *m* and M (see para 84).
(*s*) Order of merit.
(*t*) The values of *p* and P (see para 84).
(*u*) The closing error in latitude.
(*v*) ,, ,, ,, ,, longitude.
(*w*) ,, ,, ,, ,, azimuth.
(*x*) ,, ,, ,, ,, log side.
(*y*) ,, ,, ,, ,, height.

These details should be followed by a statement of the system of observation, i.e. number of zeros, measures, etc., and of the type of signals used. Any other connection with geodetic or topographical triangulation or with spirit levelling should be mentioned, and also any astronomical observations. This should be followed by a very brief statement of the sequence of the operations. Finally, complete details should be given of any irregularity or unusual procedure of observation or computation. It should be possible to assume that everything has followed the normal course except in so far as may be mentioned in this statement. Mistakes found in scrutiny should also be recorded (see para 88).

82. Permanent record.—At the end of each recess all angle books and computations should be sent to the Director for permanent record. The original and duplicate computations as detailed in paras 78 to 81, should be placed in separate folios together with the history sheet and a trace of the chart. Both original and duplicate computations must be complete in themselves.

After examination, the Director will pass the angle books and computations to the Officer in charge of the Computing Office for binding and record. The original and duplicate will be separately bound. If the series is incomplete, and if observation is being continued during the following season, the binding will be postponed and the computations will be reissued to the triangulation party during the following recess.

83. Corrections to triangulation pamphlets.—On completion of the computations, forms 20 and 21 Topo. will be prepared for newly fixed stations and intersected points respectively, and sent to the Computing Office, for incorporation in the triangulation pamphlets. It should be stated whether values given are adjusted or not. If they are unadjusted, they will be corrected by the Computing Office after adjustment has been made.

84. Criteria of accuracy.—Four quantities m, M, p and P are computed, from which is judged the accuracy of the series. m and M refer to position, p and P to heights. m and p are based solely on the accuracy of observation, while M and P take into account the lay-out of the series, and so are a better measure of the rate at which error is accumulating.

Computation of m.—m is the root-mean-square error of an unadjusted horizontal angle (in seconds), as deduced from the triangular errors:

$$m = \sqrt{\frac{\Sigma \triangle^2}{3n}},$$

where $\Sigma \triangle^2$ is the sum of the squares of all the triangular errors in the series, and n is the number of triangles. It should be computed to three decimal places: M, p and P are computed to two.

All triangles must be included in the computation of m: all four triangles of a quadrilateral must be included, not only the three which have been used in the grinding form, as it will often happen that the three best closing triangles have been selected for this purpose.

It may happen that when computing the series, a triangle or one angle of a triangle has been rejected, on the grounds that it has a large triangular error and is probably unreliable. In such cases the rejected triangle should generally be retained in the computation of m, and invariably so unless its badness is definitely attributable to some cause, which is fairly certainly known not to have affected the other triangles. The Director's approval should be obtained before excluding from the computation of m any triangle, whose three angles have been fully observed. A triangle, in which only two angles have been observed and in which the third angle has been deduced, has no closing error and obviously such a triangle is not included in the computation of m.

Computation of M.—The formula is :—

$$M = (1+f)\, m \sqrt{\frac{18}{l}},$$

where l is the average length of the sides of the triangles of the series in miles, and

$$f = \frac{1}{12} \cdot \frac{2\alpha + \gamma}{\alpha + \beta + \gamma + \delta},$$

where α, β, γ and δ are the number of simple triangles, braced quadrilaterals, pentagons and hexagons respectively, in the series. Heptagons and nonagons (if any) should be treated as pentagons: octagons and decagons as hexagons. The significance of the number 18 in the formula for M is that 18 miles is the average length of the sides of the Indian triangulation. Assuming figures to be regular, the accumulation of error of azimuth and log side is proportional to M (see Professional Paper No. 16, pages 90, 91 and 105).

Computation of p.—p is the root-mean-square error of the unadjusted difference of height between two stations (in feet), as computed from the closure of height round the three sides of a triangle.

If h_1, h_2 and h_3 are the observed differences of height in the three sides of a triangle (each normally being computed from observations at both ends of the ray), it is clear that $h_1 + h_2 + h_3$ should be zero * : let it actually equal ∇.

$$\text{Then } p = \sqrt{\frac{\Sigma \nabla^2}{3n}},$$

where n is the number of triangles.

When computing p it is only necessary to include such triangles as are essential for fixing the heights of all the stations of the series. Any very large triangles which are not essential to this purpose should be excluded: but no essential triangle, nor other triangle of average size, should be excluded merely because it closes badly.

Computation of P.—The formula for P is :—

$$P = p\sqrt{\frac{18}{l}},$$

where l is the average length of the sides of the triangles considered, in miles.

The accumulation of error of height is proportional to P (see Geodetic Report Vol. III, page 27).

85. Adjustment of series.—The adjustment of a complete network of triangulation such as that shown in fig. 1 is an operation

* When considering the signs of h_1, h_2, h_3 it is of course necessary to proceed continuously round the triangle.

which is only undertaken once or twice in the history of a survey and which need not be referred to in this handbook. The Indian triangulation has been adjusted, and the adjustment cannot be re-undertaken whenever a new series is added. The practice, when a new series is closed on to old primary triangulation at each end, is therefore to adjust it to the old work. This adjustment does not necessarily improve the accuracy of the new work: in fact it probably decreases its accuracy: but it eliminates local discrepancies which would be inconvenient to topographical triangulators. It does not preclude the possibility of a final readjustment of the whole triangulation, in which the unadjusted values would of course be employed.

It has been customary for the adjustment of the series to be carried out in the Computing Office, but there is no reason why it should not be carried out in the field party if the length of the recess, and the personnel available, make this possible.

The following is the method of adjustment which is adopted. As stated above its principal object is to avoid inconsistency, and labour is not wasted on obtaining the theoretically most probable solution. Fig. 32 represents a series which has been computed from an old side AB and closed on another CD. For the adjustment, consider the sides BAEFGHJKCD as a traverse. The closing of this traverse on to the accepted side CD, involves the satisfaction of four independent conditions, viz.:—

(*a*) Latitude of D

(*b*) Longitude of D

(*c*) Azimuth of C at D

(*d*) Log side CD.

In order to adjust the traverse BAEFGHJKCD it may be assumed that the azimuth error of each side is a small angle η'' greater than that of the preceding side, that of AB being zero. It may also be assumed that the error in the 7th figure of the log side increases from one side to the next by a small quantity ϵ: this would be an unwarrantable assumption in the case of a traverse, where the sides are directly measured, but it is quite reasonable as applied to sides of triangulation, which are each deduced from the one preceding.

The above two assumptions are of course insufficient to satisfy the four conditions, so that η and ϵ cannot be determined. To make the problem determinate the series is arbitrarily divided into two halves. In the first half the progressive increases of error in azimuth and log side are taken to be η_1 and ϵ_1 respectively, and in the second half η_2 and ϵ_2. The four conditions can now be satisfied, and the four unknowns can be determined.

The division into two separate halves should be made in the middle of the series, unless there is a marked change elsewhere in the quality of the triangulation, in which case the division should be made at the place where the quality changes.

In order to form the four equations for η_1, ϵ_1, η_2 & ϵ_2, t is necessary to compute the effect on the terminal latitude, longitude, log side and backward azimuth of unit values of :—

I. E, the change in the 7th figure of the log of the side preceding the first side of a series (viz. the change in BA).

II. ϵ, i.e. ϵ_1 and ϵ_2.

III. η, i.e. η_1 and η_2.

IV. u_0'', the change in the latitude of first station (A).

V. w_0'', the change in the backward azimuth at the first station, (viz. at A).

VI. v_0'', the change in the longitude of the first station (A).

Certain necessary multiplying factors are first worked out on form 1 Adj., the two halves of the series being kept separate (fig. 33). The suffix r indicates the serial number of the station as given in column 2, station A and the first station of the second half series being numbered 0.

The two halves of the series are next considered separately on form 2 Adj. (fig. 34). The five "cases" correspond to I-V above, VI requiring no computation, for a change in the longitude of the opening station makes equal changes in all succeeding stations. In the first half of the series cases I, IV, V and VI do not arise, and it is only necessary to compute cases II and III. In the form (2 Adj.) u_r, $v_r - v_0$ and w_r are the required changes of latitude, longitude and backward azimuth at the rth station, resulting from unit values of ϵ_1 and η_1, in cases II and III respectively. The multipliers A to H are obtained from form 1 Adj., and the other quantities are either given directly in the footnotes, or are obtained from the preceding line as indicated in the footnotes. u_r, $v_r - v_0$ and w_r are each the running sums of the preceding columns.

The last station of the first half of the series is the opening station of the second half of the series. The second half is then taken up on form 2 Adj., all five cases being considered.

The four equations can now be formed. Let prefixed suffixes 1 and 2 indicate the first and second halves of the series respectively, and let the numbers I to V indicate the "cases" so that ${}_2u_r$ III, for example, indicates the u_r obtained from case III in the second half of the series. Then at G, the end of the first half of the series.

corrn. in latitude $= {}_1u_r$ II ϵ_1 $+$ ${}_1u_r$ III $\eta_1 = u_0$ of 2nd half,

corrn. in longitude $= {}_1(v_r - v_0)$ II $\epsilon_1 + {}_1(v_r - v_0)$ III $\eta_1 = v_0$ of 2nd half,
„ „ azimuth of F $= {}_1w_r$IIϵ_1 + ${}_1w_r$III$\eta_1 = w_0$ of 2nd half,
the correction in log side FG $= r_1\,\epsilon_1$ = E of second half series.

And at D, the end of the second half of the series,

$$\delta\lambda = {}_2u_r\ \mathrm{IE} + {}_2u_r\ \mathrm{II}\epsilon_2 + {}_2u_r\mathrm{III}\eta_2 + {}_2u_r\ \mathrm{IV}\ u_0 + {}_2u_r\mathrm{V}w_0,$$

$$\delta L = v_0 + {}_2(v_r - v_0)\ \mathrm{IE} + {}_2(v_r - v_0)\ \mathrm{II}\,\epsilon_2 + {}_2(v_r - v_0)\ \mathrm{III}\ \eta_2 + {}_2(v_r - v_0)\ \mathrm{IV}\,u_0 + {}_2(v_r - v_0)\mathrm{V}w_0,$$

$$\delta\ (\text{az. of C}) = {}_2w_r\mathrm{IE} + {}_2w_r\mathrm{II}\epsilon_2 + {}_2w_r\mathrm{III}\eta_2 + {}_2w_r\mathrm{IV}u_0 + {}_2w_r\mathrm{V}w_0,$$

$$\delta\ (\log \mathrm{DC}) = r_1\,\epsilon_1 + r_2\,\epsilon_2,$$

r_1 and r_2 being the number of sides in the first and second half of the series respectively (including CD and excluding AB).

These four quantities being equated to the discrepancies (adjusted *minus* preliminary), which it is desired to adjust, provide four equations for ϵ_1, ϵ_2, η_1 and η_2. They must be solved on form 3A Adj. (fig. 35) by the usual method used for solving simultaneous equations.

In this case the equations are not symmetrical about a diagonal, and the procedure is consequently a little different from that described in para 78 in connection with the grinding of quadrilaterals. The rule is:—Multiply the first equation through by the coefficient of ϵ_1 in the second equation and then divide it by the coefficient of ϵ_1 in the first equation*: record the result with the sign changed, in the 5th line of the form, omitting the coefficient of ϵ_1. Then multiply the first equation through by the coefficient of ϵ_1 in the third equation, divide it by the coefficient of ϵ_1 in the first, and record it with the sign changed in the 6th line of the form. Again multiply the first equation by the coefficient of ϵ_1 in the fourth, divide by the coefficient of ϵ_1 in the first, and record it in the 7th line. Then add together the 2nd and 5th, 3rd and 6th, and 4th and 7th lines of the form and record their sums in the 8th, 9th and 10th lines respectively. These three lines then represent three equations for η_1, ϵ_2 and η_2, which are reduced to two, and eventually to one equation, in a similar manner. It will be seen that the rule is the same as that given for the grinding of quadrilaterals, except that the latter is simplified by the symmetry about a diagonal.

Five decimals should normally be kept in the computation, and the results should be substituted in the equations to verify that the solution is correct.

If in any equation there is a coefficient of less than 0·1, it will be convenient to multiply that equation through by 10 or 100 before solution, so as to increase the smallest coefficient to more than 0·1.

* In practice it is of course more convenient first to find the quotient of the two coefficients and then to multiply the equation by this quotient.

The adjusted log sides of the adjusted flank are then immediately derived from the values of ϵ. The resulting change of latitude, longitude and azimuth at each station is obtained successively on form 5 Adj. On this form $\delta s/s$ is the fractional change in the side considered, $\delta s/s$ cosec $1''$ being equal to 0·0475 of the change in the 7th decimal of the log side : the factor f is unity in this case : u' and v' are the changes of latitude and longitude of the preceding station.

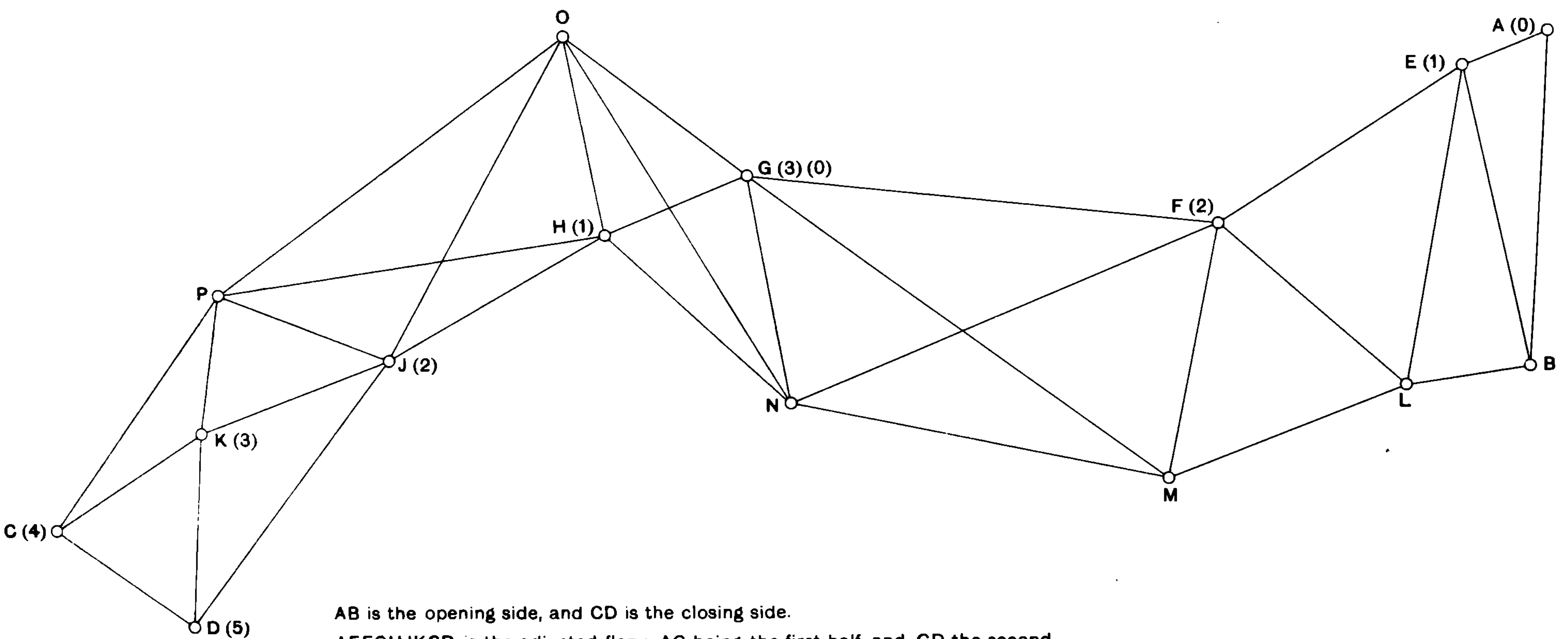
This computation should be carried up to the two closing stations : if their latitudes and longitudes are correctly reproduced, the whole process is checked.

It is next necessary to adjust the remaining stations of the series, and to find the adjusted lengths and azimuths of the sides joining them. To fix each station two arbitrary assumptions have to be made, which take the following form :— For each station which remains to be adjusted, consider a triangle of which it is the apex and the nearest " traverse side " is the base. Then the changes in the base angles of these triangles are assumed to bear the same ratio to η_1 (or η_2) as the base angles themselves bear to the angles of the " traverse ". Thus to adjust the station L we consider the triangle FEL and we assume that the changes in the angles FEL and EFL are $f\eta_1$ and $f_1\eta_1$ respectively, where $f=\frac{\text{FEL}}{\text{FEA}}$ and $f_1 = \frac{\text{EFL}}{\text{EFG}}$. It must be noted that these assumptions are arbitrary, and that consequently only two such assumptions can be made for each station to be fixed: having made sufficient assumptions, it cannot be assumed that the corrections to any other angles (e.g. AEB) are equal to the corresponding $f\eta_1$.

The resulting changes in the sides of the triangles associated with each unfixed station are then computed on form 4 Adj. (fig. 36) on which should be set up sufficient (and no more than sufficient) triangles to fix the remaining stations. The changes in latitude, longitude and reverse azimuth at each of these stations are then computed on form 5 Adj. (fig. 37), each station being computed from both sides of the triangle by which it is being fixed. The two sides should of course give accordant results for the changes of latitude and longitude.

The triangulation pamphlets publish the azimuths and log lengths of all the sides of geodetic triangulation. It is therefore necessary to compute the changes in these quantities in any sides not already dealt with. This is done on form 6 Adj. (fig. 38).

Fig. 32



AB is the opening side, and CD is the closing side.
AEFGHJKCD is the adjusted flank, AG being the first half, and GD the second.
The numbers in brackets are the serial numbers of the stations used in the adjustment computations.

Fig. 33

1 Adj. **Survey of India** Page

No. 15 PARTY (Triangulation) SEASON 1928-30.

Name of Series Chittagong

Computation of Factors A, B, C, D, E, F, G and H for use on 2 Adj.

Names of stations in "Traverse"	Serial number = r	$\Delta\lambda_r = \lambda_r - \lambda_{r-1}$	$\Delta L_r = L_r - L_{r-1}$	Circular Measure to 5 places of decimals: $\Delta\lambda_r = A$	Circular Measure to 5 places of decimals: $\Delta L_r = B$	λ_{r-1}	To four places of decimals: $\sin\lambda_{r-1}$	$\cos\lambda_{r-1}$	$\tan\lambda_{r-1}$	$\sec\lambda_{r-1}$	To five places of decimals: $A \times \tan\lambda_{r-1} = C$	$A \times \sec\lambda_{r-1} = D$	$B \times \sin\lambda_{r-1} = E$	$B \times \cos\lambda_{r-1} = F$	$B \times \tan\lambda_{r-1} = G$	$B \times \sec\lambda_{r-1} = H$
		′ ″	′ ″			° ′	First		Half							
Waibula H.S.	0															
Yetagong H.S.	1	-02 08	-06 05	-0.00062	-0.00177	23 02	0.3913	0.9203	0.4252	1.0866	-0.00026	-0.00067	-0.00069	-0.00163	-0.00075	-0.00192
Zemuklang H.S.	2	-10 08	-17 10	-0.00295	-0.00499	23 00	0.3907	0.9205	0.4245	1.0864	-0.00125	-0.00320	-0.00195	-0.00459	-0.00212	-0.00542
Lungleng H.S.	3	+03 10	-32 59	+0.00092	-0.00959	22 50	0.3881	0.9216	0.4211	1.0850	+0.00039	+0.00100	-0.00372	-0.00884	-0.00404	-0.01041
	4	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
	5	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
	6	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
	7	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
	8	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
							Second		Half							
	9	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
Lungleng H.S.	0	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
Sairep H.S.	1	-03 48	-10 09	-0.00111	-0.00295	22 53	0.3889	0.9213	0.4221	1.0854	-0.00047	-0.00120	-0.00115	-0.00272	-0.00125	-0.00320
Mullianphui H.S.	2	-08 15	-15 28	-0.00240	-0.00450	22 49	0.3878	0.9218	0.4207	1.0849	-0.00101	-0.00260	-0.00175	-0.00415	-0.00189	-0.00488
Phukamoin H.S.	3	-04 36	-13 21	-0.00134	-0.00388	22 41	0.3856	0.9227	0.4180	1.0838	-0.00056	-0.00145	-0.00150	-0.00358	-0.00162	-0.00421
Sitapahār H.S.	4	-06 35	-10 34	-0.00192	-0.00307	22 36	0.3843	0.9232	0.4163	1.0832	-0.00080	-0.00208	-0.00118	-0.00283	-0.00128	-0.00333
Gilachhari H.S.	5	-06 25	+09 50	-0.00187	+0.00286	22 29	0.3824	0.9240	0.4139	1.0823	-0.00077	-0.00202	+0.00109	+0.00264	+0.00118	+0.00310
	6	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0
	7	-	-	-0.0	-0.0		0.	0.	0.	1.	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0

Computed and Compared by A. K. M. and C. B. M. *Date* 16. 12. 30.

G.B.P.O.—1931.

2 Adj. **Survey of India** Fig. 34

Page

No. 15 PARTY (Triangulation) SEASON 1928-30.

Name of series Chittagong

Computation of Coefficients for Solution of Equations.

(1) Case	(2) r	(3) k* × A‖	(4) − m† × F‖	(5) (3) + (4) = $u_r - u_{r-1}$	(6) ‡ u_r	(7) k* × B‖	(8) m† × D‖	(9) u_{r-1} G‖	(10) (7) + (8) + (9) = $v_r - v_{r-1}$	(11) $v_r - v_0$	(12) k* × E‖	(13) m† × C‖	(14) u_{r-1} H‖	(15) (12)+(13)+(14) = $w_r - w_{r-1} - \eta$ ¶	(16) § w_r
First Half II	0	k = r	m = W_{r-1}		u_0 = 0	k = r	m = W_{r-1}				k = r	m = W_{r-1}		η = 0	w_0 = 0
	1	-0·00062	-0·00000	-0·00062	-0·00062	-0·00177	-0·00000	-0·00000	-0·00177	-0·00177	-0·00069	-0·00000	-0·00000	-0·00069	-0·00069
	2	-0·00590	-0·00000	-0·00590	-0·00652	-0·00998	-0·00000	+0·00000	-0·00998	-0·01175	-0·00390	+0·00000	+0·00000	-0·00390	-0·00459
III	0	k = 0	m = W_{r-1}+1		u_0 = 0	k = 0	m = W_{r-1}+1				k = 0	m = W_{r-1}+1		η = 1	w_0 = 0
	1	-0·00000	+0·00163	+0·00163	+0·00163	-0·00000	-0·00067	-0·00000	-0·00067	-0·00067	-0·00000	-0·00026	-0·00000	-0·00026	+0·99974
	2	-0·00000	+0·00918	+0·00918	+0·01081	-0·00000	-0·00640	-0·00000	-0·00640	-0·00707	-0·00000	-0·00250	-0·00001	-0·00251	+1·99723
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
Second Half I	0	k = 1	m = W_{r-1}		u_0 = 0	k = 1	m = W_{r-1}				k = 1	m = W_{r-1}		η = 0	w_0 = 0
	1	-0·00111	-0·00000	-0·00111	-0·00111	-0·00295	+0·00000	+0·00000	-0·00295	-0·00295	-0·00115	+0·00000	+0·00000	-0·00115	-0·00115
	2	-0·00240	-0·00000	-0·00240	-0·00351	-0·00450	+0·00000	+0·00000	-0·00450	-0·00745	-0·00175	+0·00000	+0·00001	-0·00174	-0·00289
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
II	0	k = r	m = W_{r-1}		u_0 = 0	k = r	m = W_{r-1}				k = r	m = W_{r-1}		η = 0	w_0 = 0
	1	-0·00111	-0·00000	-0·00111	-0·00111	-0·00295	+0·00000	+0·00000	-0·00295	-0·00295	-0·00115	+0·00000	+0·00000	-0·00115	-0·00115
	2	-0·00480	-0·00000	-0·00480	-0·00591	-0·00900	+0·00000	+0·00000	-0·00900	-0·01195	-0·00350	+0·00000	+0·00001	-0·00349	-0·00464
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
III	0	k = 0	m = W_{r-1}+1		u_0 = 0	k = 0	m = W_{r-1}+1				k = 0	m = W_{r-1}+1		η = 1	w_0 = 0
	1	-0·00000	+0·00272	+0·00272	+0·00272	-0·00000	-0·00120	-0·00000	-0·00120	-0·00120	-0·00000	-0·00047	-0·00000	-0·00047	+0·99953
	2	-0·00000	+0·00830	+0·00830	+0·01102	-0·00000	-0·00520	-0·00001	-0·00521	-0·00641	-0·00000	-0·00202	-0·00001	-0·00203	+1·99750
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
IV	0	k = 0	m = W_{r-1}		u_0 = 1	k = 0	m = W_{r-1}				k = 0	m = W_{r-1}		η = 0	w_0 = 0
	1	-0·00000	-0·00000	-0·00000	+1·00000	-0·00000	-0·00000	-0·00125	-0·00125	-0·00125	-0·00000	-0·00000	-0·00320	-0·00320	-0·00320
	2	-0·00000	-0·00001	-0·00001	+0·99999	-0·00000	+0·00001	-0·00189	-0·00188	-0·00313	-0·00000	+0·00000	-0·00488	-0·00488	-0·00808
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
V	0	k = 0	m = W_{r-1}		u_0 = 0	k = 0	m = W_{r-1}				k = 0	m = W_{r-1}		η = 0	w_0 = 1
	1	-0·00000	+0·00272	+0·00272	+0·00272	-0·00000	-0·00120	-0·00000	-0·00120	-0·00120	-0·00000	-0·00047	-0·00000	-0·00047	+0·99953
	2	-0·00000	+0·00415	+0·00415	+0·00687	-0·00000	-0·00260	-0·00001	-0·00261	-0·00381	-0·00000	-0·00101	-0·00001	-0·00102	+0·99851
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·
		-0·	-0·0	-0·	- ·	- ·	-0·0	-0·0	-0·	- ·	-0·	-0·0	-0·0	-0·	- ·

* k = 1 for Case I, k = r for Case II, k = 0 for Cases III, IV, V.
† m = w_{r-1} for Cases I, II, IV, V: m = w_{r-1} + 1 for Case III.

‡ u_0 = 0 for Cases I, II, III, V: u_0 = 1 for Case IV.
§ w_0 = 0 for Cases I, II, III, IV: w_0 = 1 for Case V.

‖ A, B, C, D, E, F, G and H are to be taken from 1 Adj.
¶ η = 0 except in case III where η = 1.

Computed and Compared by A. K. M. and C. B. M. *Date* 18 12. 30.

G.B. P.O.—1931

Note:- In order to save space, the illustration shows only two stations in each case.

Fig. 35

3A Adj. **Survey of India** Page

No. 15 PARTY (Triangulation) SEASON 1928-30 .

Name of Series Chittagong

Solution of four Simultaneous Equations for Determination of $\epsilon_1, \eta_1, \epsilon_2, \eta_2$

	Coefficients of				e
	ϵ_1	η_1	ϵ_2	η_2	
1	− 0.2992	+ 0.6921	− 0.2697	+ 0.1988	− 1.50
2	− 0.7485	− 0.3232	− 0.2152	− 0.2916	− 2.97
3	− 0.2898	+29.8691	− 0.0839	+49.8876	+17.5
4	+ 3.0000	...	+ 5.0000	...	+ 5.70
5		− 1.7314	+ 0.6747	− 0.4973	+ 3.75251
6		− 0.6704	+ 0.2612	− 0.1926	+ 1.45287
7		+ 6.9395	− 2.7042	+ 1.9933	−15.04011
8		− 2.0546	+ 0.4595	− 0.7889	+ 0.78251
9		+29.1987	+ 0.1773	+49.6950	+18.95287
10		+ 6.9395	+ 2.2958	+ 1.9933	− 9.34011
11			+ 6.5301	−11.2114	+11.12055
12			+ 1.5520	− 2.6645	+ 2.64296
13			+ 6.7074	+38.4836	+30.07342
14			+ 3.8478	− 0.6712	− 6.69715
15				−22.0765	−17.25192
16				−22.7477	−23.94907

$\eta_2 = +1.0528$

$+6.7074\,\epsilon_2 = -38.4836\,\eta_2 + 30.07342$

$= -40.51553 + 30.07342 = -10.44211$

$\epsilon_2 = -1.5568$

$-2.0546\,\eta_1 = -0.4595\,\epsilon_2 + 0.7889\,\eta_2 + 0.78251$

$= +0.71535 + 0.83055 + 0.78251 = +2.32841$

$\eta_1 = -1.1333$

$-0.2992\,\epsilon_1 = -0.6921\,\eta_1 + 0.2697\,\epsilon_2 - 0.1988\,\eta_2 - 1.50$

$= +0.78436 - 0.41987 - 0.20930 - 1.50 = -1.34481$

$\epsilon_1 = +4.4947$

CHECK

Substitution of values of ϵ_1, η_1, ϵ_2 and η_2 in the four Original Equations.

ϵ_1	η_1	ϵ_2	η_2	Sum	Corrections	Residuals
− 1.3448	− 0.7844	+ 0.4199	+ 0.2093	− 1.500	− 1.50	0.000
− 3.3643	+ 0.3663	+ 0.3350	− 0.3070	− 2.970	− 2.97	0.000
− 1.3026	−33.8507	+ 0.1306	+52.5217	+ 17.499	+ 17.5	0.001
+13.4841	...	− 7.7840	...	+ 5.700	+ 5.70	0.000

Computed and Compared by A. K. M. and R. B. S. *Date* 16. 6. 31.

Survey of India

No. 15 PARTY (Triangulation) SEASON 1928-30.

Name of Series Chittagong

Computation of Corrections to two other sides when the correction to base is known.

Station A	Yetagong H.S.	Zemuklang H.S.								
„ B	Zemuklang •	Lungleng •								
„ C	Mongklang •	Zovailangklang •								
Angle A	47° 23′ 39″	84° 46′ 43″								
cot A	+ 0·92	+ 0·091	–	–	–	–	–	–	–	–
$f^{*}\eta$	– 0·32	– 0·44	–	–	–	–	–	–	–	–
$f\eta$ cot A = (1)	– 0·29	– 0·04	–	–	–	–	–	–	–	–
Angle B	73° 48′ 46″	30° 13′ 44″								
cot B	+ 0·29	+ 1·7	–	–	–	–	–	–	–	–
$f_1^{*}\eta$	– 0·38	+ 0·21	–	–	–	–	–	–	–	–
$f_1\eta$ cot B = (2)	– 0·11	+ 0·36	–	–	–	–	–	–	–	–
Angle C	58° 47′ 35″	64° 59′ 33″								
cot C	+ 0·61	+ 0·47	–	–	–	–	–	–	–	–
$(f+f_1)\eta$	– 0·70	– 0·23	–	–	–	–	–	–	–	–
$(f+f_1)\eta$ cot C = (3)	– 0·43	– 0·11	–	–	–	–	–	–	–	–
$r\epsilon$	+ 8·99	+ 13·48	–	–	–	–	–	–	–	–
$r\epsilon$ + (3) = (4)	+ 8·56	+ 13·37	–	–	–	–	–	–	–	–
(4) + (1) = $\frac{\delta a}{a}$ cosec 1″ = (5)	+ 8·27	+ 13·33	–	–	–	–	–	–	–	–
(4) + (2) = $\frac{\delta b}{b}$ cosec 1″ = (6)	+ 8·45	+ 13·73	–	–	–	–	–	–	–	–
Correction to log a (BC) = (5) × 2·105 × 10^{-6}	+ 0·0000174	+ 0·0000281	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000
Correction to log b (AC) = (6) × 2·105 × 10^{-6}	+ 0·0000178	+ 0·0000289	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000	– 0·000

* f, f_1 are fractional factors for η, the change of angle.

Computed and Compared by A. K. M. and R. B. S. *Date* 16. 6. 31.

G.B.P.O.—1931.

Fig. 37

5 Adj. Survey of India Page

No. 15 PARTY (Triangulation) SEASON 1928-30.

Name of Series Chittagong

Traverse line Waibula H.S. to Lungleng H.S. (1st half), Lungleng H.S. to Gilachhari H.S. (2nd half)

Computation of Corrections to Lat., Long. and Azimuth at end of a ray whose Length and Azimuth have been corrected.

Station A	Waibula H.S.	Yetagong H.S.	Zemuklang H.S.		Lungleng H.S.	Sairep H.S.				
Station B	Yetagong "	Zemuklang "	Lungleng "		Sairep "	Mullianphui "				
$\frac{\xi}{\rho}$ cosec 1″ = a	+ 4″·49	+ 8″·99	+ 13″·48	— ″.	+ 11″·93	+ 10″·31	— ″.	— ″.	— ″.	— ″.
*w′ + fη = β	− 1″·13	− 2″·26	− 3″·41	— ″.	− 2″·41	− 1″·37	— ″.	— ″.	— ″.	— ″.
Radian { $\lambda_B - \lambda_A = \Delta\lambda$	−0·00062	−0·00295	+0·00092	−0·0	−0·00111	−0·00240	−0·0	−0·0	−0·0	−0·0
$L_B - L_A = \Delta L$	−0·00177	−0·00499	−0·00959	−0·0	−0·00295	−0·00450	−0·0	−0·0	−0·0	−0·0
$Az_B - Az_A = \Delta A$	−0·00069	−0·00194	−0·00373	−0·0	−0·00114	−0·00174	−0·0	−0·0	−0·0	−0·0
Latitude of Stn. A	23° 01′ 48″	22° 59′ 40″	22° 49′ 32″	° ′ ″	22° 52′ 42″	22° 48′ 54″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
sin λ of Stn. A	0·3912	0·3906	0·3879	0·	0·3888	0·3878	0·	0·	0·	0·
cos λ "	0·9203	0·9205	0·9217	0·	0·9213	0·9218	0·	0·	0·	0·
sec λ "	1·0866	1·0863	1·0850	1·	1·0854	1·0849	1·	1·	1·	1·
tan λ "	0·4251	0·4244	0·4209	0·	0·4220	0·4207	0·	0·	0·	0·
sin λ × cos λ = c	0·3600	0·3595	0·3575	0·	0·3582	0·3575	0·	0·	0·	0·
a.Δλ	− 0·0028	− 0·0265	+ 0·0124	− 0·	− 0·0132	− 0·0249	− 0·	− 0·	− 0·	− 0·
− cos λ . ΔL.β	− 0·0018	− 0·0104	− 0·0301	− 0·	− 0·0065	− 0·0057	− 0·	− 0·	− 0·	− 0·
Sum = u − u′	− 0·0046	− 0·0369	− 0·0177	− 0·	− 0·0197	− 0·0306	− 0·	− 0·	− 0·	− 0·
Corrn. to λ_A = u′	− 0·0000	− 0·0046	− 0·0415	− ·	− 0·0592	− 0·0789	− ·	− ·	− ·	− ·
Sum = u = Corrn. to λ_B	− 0·0046	− 0·0415	− 0·0592	− ·	− 0·0789	− 0·1095	− ·	− ·	− ·	− ·
a.ΔL	− 0·0079	− 0·0449	− 0·1293	− 0·	− 0·0352	− 0·0467	− 0·	− 0·	− 0·	− 0·
sec λ. Δλ.β	+ 0·0008	+ 0·0072	− 0·0034	− 0·	+ 0·0029	+ 0·0036	− 0·	− 0·	− 0·	− 0·
tan λ. u′.ΔL	+ 0·0000	+ 0·0000	+ 0·0002	− 0·	+ 0·0001	+ 0·0001	− 0·	− 0·	− 0·	− 0·
Sum = v − v′	− 0·0071	− 0·0377	− 0·1325	− 0·	− 0·0322	− 0·0430	− 0·	− 0·	− 0·	− 0·
Corrn. to L_A = v′	− 0·0000	− 0·0071	− 0·0448	− ·	− 0·1773	− 0·2095	− ·	− ·	− ·	− ·
Sum = v = Corrn. to L_B	− 0·0071	− 0·0448	− 0·1773	− ·	− 0·2095	− 0·2525	− ·	− ·	− ·	− ·
u′ ÷ c = d	− 0·00	− 0·01	− 0·12	− ·	− 0·17	− 0·22	− ·	− ·	− ·	− ·
a.ΔA (To 2 places of decimal)	− 0·00	− 0·02	− 0·05	− 0·0	− 0·01	− 0·02	− 0·0	− 0·0	− 0·0	− 0·0
tan λ. Δλ.β	+ 0·00	+ 0·00	− 0·00	− 0·0	+ 0·00	+ 0·00	− 0·0	− 0·0	− 0·0	− 0·0
d. ΔA	+ 0·00	+ 0·00	+ 0·00	− 0·0	+ 0·00	+ 0·00	− 0·0	− 0·0	− 0·0	− 0·0
Sum = w − β	− 0·00	− 0·02	− 0·05	− 0·0	− 0·01	− 0·02	− 0·0	− 0·0	− 0·0	− 0·0
β	− 1·13	− 2·26	− 3·41	− .	− 2·41	− 1·37	− ·	− ·	− ·	− ·
w = Corrn. to Az. of A	− 1·13	− 2·28	− 3·46	− .	− 2·42	− 1·39	− ·	− ·	− ·	− ·

* w′ = Correction to back azimuth of traverse side.

Computed and Compared by A. K. M. and R. B. S. *Date* 16 . 6 . 31 .

Geod. Br. P.O.—1931.

Fig. 38

6 Adj. Survey of India Page

No. 15 PARTY (Triangulation) SEASON 1928-30.

Name of Series Chittagong

Computation of Corrections to Azimuth and log side of a ray joining two points whose co-ordinates are slightly altered.

Station A	Wone-lone-taung H.S.	Wone-lone-taung H.S.							
„ B	Yetagong H.S.	Mongklang H.S.							
$u - u'$	− 0·0046	− 0·0579	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
$\Delta\lambda = \lambda_B - \lambda_A$	+ 0° 19′ 38″	− 0° 01′ 12″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″
$\Delta\lambda$ in radian	+ 0·00571	− 0·000349	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0
$u - u'/\Delta\lambda$ in radian	− 0·806	+ 165·903	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$v - v'$	− 0·0071	− 0·0042	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
$\Delta L = L_B - L_A$	− 0° 04′ 35″	− 0° 08′ 36″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″	− ° ′ ″
ΔL in radian	− 0·00133	− 0·00250	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0	− 0·0
$v - v'/\Delta L$ in radian	+ 5·338	+ 1·680	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
λ' = Lat. of stn. A	22° 40′ 02″	22° 40′ 02″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
$\tan \lambda'$	0·418	0·418	0·	0·	0·	0·	0·	0·	0·	0·
u'	− 0·000	− 0·000	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
$\tan \lambda' \times u'$	− 0·000	− 0·000	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·	− 0·
Az. at A of B = A′	167° 48′ 11″	81° 28′ 38″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
$2 \text{ cosec } 2A' = a$	− 4·843	+ 6·823	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$2 \cot 2A' = \beta$	− 4·410	− 6·523	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$\frac{v-v'}{\Delta L} - \frac{u-u'}{\Delta\lambda} - \tan\lambda' u' = (1)$	+ 6·144	− 164·223	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$(1) \div a = w'$ *	− 1·27	− 24·07	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$\frac{v-v'}{\Delta L} + \frac{u-u'}{\Delta\lambda} - \tan\lambda' u' = (2)$	+ 4·332	+ 167·583	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$w' \times \beta = (3)$	+ 5·601	+ 157·009	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$(2) - (3) = 2\frac{\delta s}{s} \text{ cosec } 1''$	− 1·069	+ 10·574	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
$\frac{\delta s}{s} \text{ cosec } 1''$	− 0·53	+ 5·29	− ·	− ·	− ·	− ·	− ·	− ·	− ·	− ·
Correction to log side $= \frac{\delta s}{s} \text{ cosec } 1'' \times 2 \cdot 105 \times 10^{-6}$	− 0·0000011	+ 0·0000111	− 0·000	− 0·000	− 0·000	− 0·000	− 0·000	− 0·000	− 0·000	− 0·000

Note:—Dashed letters refer to A station.

* w' = Correction to azimuth BA.

Computed and Compared by A. K. M. and R. B. S. *Date* 16. 6. 31.

G.B.P.O.—1931.

A full synopsis is then prepared on form 21 Trian. and forms 20 and 21 Topo. are corrected.

The positions of any intersected points of the series should be adjusted on 4 and 5 Adj. in the same way as non-traverse stations. No form 6 Adj. is necessary.

86. Miscellaneous computations.—*Satellite stations.*—The reduction of observations to and from a satellite station is explained in the Handbook of Topography, Chapter III. The necessary computations, illustrated by a diagram, will be carried out in the field in the angle book (see para 63). Provided one satellite has been consistently used, the observed angles, uncorrected, will be entered in the abstract form, where the general mean will be taken out in the usual way, and then the corrected angle will be entered in red beneath it. This corrected mean will be used in all subsequent computations, except for the analysis of graduation error and computation of weights. But if more than one satellite has been used in the measurement of any angle, or if some observations have been made direct, the satellite corrections must be applied before making the abstract.

When considering the number of decimals required in the satellite reduction, the computer may be guided by the fact that one foot subtends an angle of one second at a distance of 39 miles.

Form 12 Trian.—An intersected point may be observed from two stations which are not joined by an observed ray. In order to form a triangle for the computation of such an intersected point, it is necessary to compute the mutual distance and azimuths of the two stations. This can be done on form 12 Trian., the data being their latitudes and longitudes. The quantities R,′ S,′ etc. are obtained from the Auxiliary Tables, 4th edition, in the Explanation of which an example is given.

If the latitudes and longitudes are only given to 3 decimals of a second, the log side cannot be expected to be correct to the 7th decimal. In the case of short sides, heavy discrepancies may be expected.

Form 18 Trian.—As an alternative to form 12 Trian. it will sometimes be possible to use form 18 Trian., on which the required base can be deduced if it forms part of a triangle of which the other two sides and the included angle are known. This form is shorter than 12 Trian., and should be used in preference to it whenever possible.

It must be remembered that this form deals with the angles of the equivalent plane triangle, a plane triangle whose sides are equal to that of the spheroidal triangle. Consequently one third of the spherical excess must be deducted from the spherical value of the given included angle, and it must be added to the deduced values of the other two angles.

If the known angle is small, and if it is only known to the nearest 0″·01 some inaccuracy may occur in the 7th figure of the deduced log side.

As stated in para 14 a geodetic series should not ordinarily be based on a side which has been computed in this way. Forms 12 and 18 Trian. are computationally correct, and if such a procedure is inevitable, they can be used. The objection is that the distance between two disconnected points is naturally less precisely determined than the distance between two points which form the side of a triangle.

87. Precautions against errors in computations.— It should be unnecessary to state that accuracy in the computations is essential. All the care expended on a series, which may have cost one or two lakhs of rupees to observe, may be thrown away if a single computational error goes undetected. The precautions against computational error take three forms:—

(*a*) Computation in duplicate.

(*b*) The provision of automatic checks in the computation.

(*c*) Final scrutiny.

Computation in duplicate.—This is not a complete check on accuracy: there are certain points in any computation which entail special risk of error, and sooner or later both computers may make the same mistake. An example of such a point is when a log cosine appears in a form among a number of log sines: both computers may forget that in a table of log cosines the differences are negative between 0° and 90°. With skilled computers, accustomed to the form on which they are working and each making very few errors, duplicate computation is a fair check, but with unskilled computers whose mistakes involve frequent comparison it is a very poor check indeed. This is especially the case when one computer is much less skilled than the other: as a result of continually expecting to find himself wrong, he may find an imaginary error in his own work when in fact he chances to be correct.

The foregoing remarks must not be understood to condone inaccurate computations. Trained computers, working in duplicate, are open to heavy censure if subsequent scrutiny reveals error in their work.

Comparison between original and duplicate should be as infrequent as is compatible with the avoidance of undue waste of time on account of undetected errors.

Automatic checks.— A much more reliable check is that in which identical results have to be obtained by means of two completely different pieces of arithmetic. Such a check is an almost certain proof of the correctness of all stages of the computation in which an error would spoil the agreement, provided the check is obtained without too much searching for mistakes in the preceding work. If agreement is only obtained after correcting a dozen errors, it fails to be certain proof of accuracy: it becomes conceivable that the agreement has come about by chance.

Checks of this nature are provided whenever possible, as for instance in the computation of latitudes and longitudes, which are entirely self-checking. Self-checking computations should always be done in duplicate, in the same way as other computations. Not only is it an additional precaution, but it will often actually save time by keeping the computations correct at intermediate stages: a computer who fails to get his checks right may spend much more time in finding his errors than he spends in the original computations.

88. Scrutiny of computations.—The completed computations must be scrutinised by a responsible officer, preferably the Officer in charge of the party, and in any case of rank not less than that of the Upper Subordinate Service. As the result of his scrutiny this officer must be able to say that he *knows* the computation to contain no serious error, apart altogether from his confidence in the skill of the computers. In this context a "serious error" does not include an error of a few tenths of a second in azimuth, nor of a few units in the 7th figure of a log side, nor does it apply in detail to the weighting of the angles, which is inevitably very doubtfully accurate.

The name of the officer responsible for the scrutiny appears in the history sheet (see para 81). Unless it is otherwise mentioned, it is implied not only that he vouches for the accuracy of the computations, but also that (where applicable) he has specifically carried out the procedure detailed below.

Angle book.—The scrutinising officer need not check the means in the angle books: any error large enough to make a serious change in the general mean will stand out by reason of its discrepancy from the other measures. He should see that the angle books have been signed by the observer and recorder, and that they have been compared with the duplicate. He also should verify the reduction of satellite stations. This verification should not take the form of checking through the figures given, but should be done graphically (see fig. 39). The station and its satellite are drawn to a suitable scale and the rays are shown at their correct azimuths, the rays to stations and their satellites being of course assumed parallel. The perpendicular distances BP, BQ, BR, BS are scaled off the diagram and are roughly converted into angular correction by the rule:—

$$\text{Angle} = \frac{\text{Perpendicular}}{\text{Length of ray}} \times \text{cosec } 1'' \textit{ seconds.}$$

This check need only be carried out sufficiently accurately to serve as a rough check. Time need not be spent in reconciling differences of a few tenths of a second. Particular attention should be paid to the sign of the correction.

The bubble correction to vertical angles should be checked in a few cases, and also the sign (elevation or depression) of any vertical angles which are very nearly zero. It should be verified that descriptions of stations are adequate.

Abstracts.—The copying of the abstract from the angle book need not be examined, but the form should be looked through and any exceptionally wide measures should be examined, both as regards their copying from the angle book and as regards their meaning in the angle book. One single measure of each angle should in any case be checked through from the original observations to ensure that the degrees and minutes have been correctly taken out.

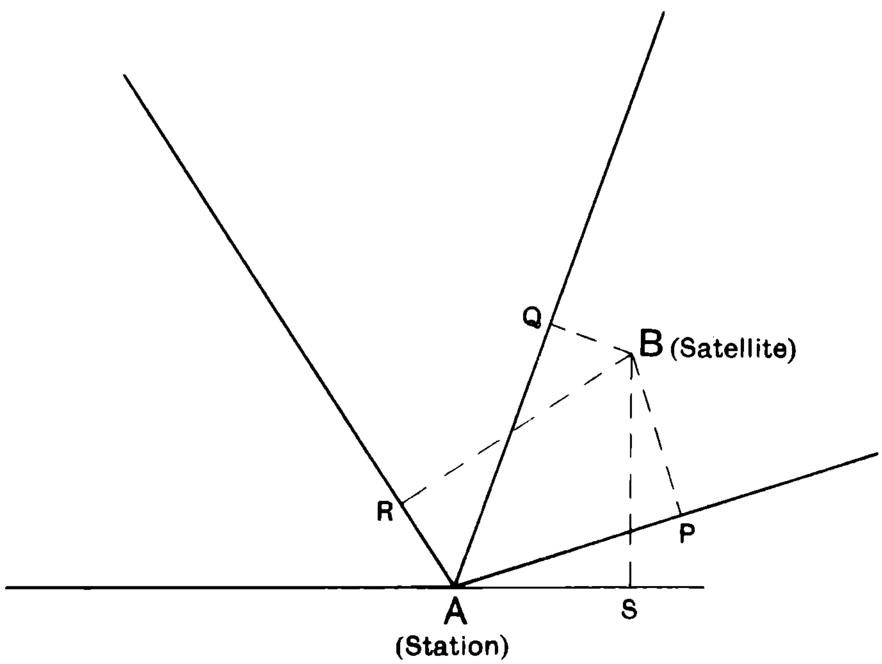
The zero means need not be fully checked, but a glance at the individual measures on each zero will serve to show that the zero mean is correct within 1 or 2 seconds at most. If the mean looks wrong, it should be checked. Any less error will not have any great effect on the general mean.

The checking of the general mean calls for more care. The number of zero means greater than the general mean should be counted: if it is 4, 5 or 6 in the case of 10 zeros (5, 6 or 7 with 12 zeros) the mean may be accepted. If the distribution of large and small values is less equal, the mean should be checked.

Analysis of graduation error.—As this is only required for the computation of weights, rigorous scrutiny can largely be dispensed with. A few of the entries in the classification table (fig. 26) should be compared with the angle book, to see that they are in their correct squares. Any isolated entries separated from the majority of the other entries, should be looked at particularly carefully. The subsequent computation of the smoothed graduation error should be followed through in the case of one square in the second table (fig. 25) as far as the drawing of the curve showing the graduation error. This is to verify that the computers have correctly understood the process. Any exceptionally large graduation errors should be verified by finding the entries responsible for them in the classification table.

Weights.—If the systematic graduation error has been found to be appreciable, the scrutinising officer should verify that its application results in the corrected zero means in the 5th column of form 7A Trian. being generally more uniform than the uncorrected means in the 3rd column, as a check on the corrections having been applied with the right sign.

Fig. 39



The only further scrutiny required is to examine the zero means of any angles whose weights have come out exceptionally high or exceptionally low, and to see that the inconsistency of their zero means is in general accord with the weights deduced. It should be remembered that the reciprocal weight will vary roughly as the square of the range of the zero means.

Spherical excess.—The spherical excess should be checked by Table 1 Sur., the area of each triangle being taken from a map on such a scale as $\frac{1}{4}$ inch = 1 mile.

Grinding.—The forms should be examined to see that the tests have proved. This effectually checks the working of the forms.

Computation of log sides.—It should be verified that the angles used for computation add up to 180°, that all possible triangles have been set up, and that the common sides have proved within 3 in the last (7th) decimal. This completely checks the setting up and computation of all figures except simple triangles. In the case of a simple triangle the copying of the observed angles from the abstract form should be checked, and the distribution of triangular error should be roughly examined to see that the worst angles are given the largest corrections. It is not necessary to check the log sines etc., since their accuracy is proved by the agreement of the two deductions in form 13A Trian.

The copying of the log side of the first triangle in each figure should be verified, for there is no other check on it.

Latitudes and longitudes.—The scrutinising officer should verify that the deductions are properly made in pairs (see para 78) and that the results agree, and that the two azimuths at station C differ by the accepted value of the spherical angle at C. This completely checks the computation and setting up of this form, and also the computation of log sides on form 11 Trian.

Heights.—The height of every station is obtained by angles observed at both ends of at least two rays. The mutual agreement of these four determinations is a good check against gross error in the computation. The scrutinising officer should check through at least two* computations (one elevation and one depression) with special reference to the signs of the various small corrections. The computations selected for check should be those which are most discordant.

Intersected points.—Intersected points should be examined in the same way as stations.

* i. e. two in the season's work. Not two per station.

Synopsis.—He should check the synopsis from forms 13 A and 16 Trian. and also the corrections to the triangulation pamphlet (forms 20 and 21 Topo.) He should verify that action has been taken to obtain the correct spelling of station names, and that the history sheet and chart are properly prepared.

m, M, p and P.—He should verify the computation of m and p by the approximate formulæ:—

$$m \doteq \frac{\cdot 845}{\cdot 674\sqrt{3}} \frac{\Sigma|\Delta|}{n} = 0{\cdot}72 \times (\text{Average triangular error}), \text{ where}$$

$\Sigma|\Delta|$ is the sum of all the triangular errors without regard to sign, and n is the number of triangles.

Similarly $p = 0{\cdot}72 \times$ (Average triangular closure of height).

The derivation of M and P should be roughly checked.

Adjustment.—The adjustment of the selected flank is completely checked by computing the adjusted latitude etc. of each station in turn from the preceding station, and seeing that the latitudes etc. of the last station agree with the values to be accepted. The scrutinising officer has only to see that this has been done, and that the values to be accepted have been correctly abstracted. The computation of the latitudes and longitudes of the remaining stations on forms 4 and 5 Adj. is self-checking provided deductions on form 5 Adj. are set up in pairs as described in para 85. He should verify that this has been done and that the checks have proved. Two or three deductions on form 6 Adj. should be checked by computation on forms 12 or 18 Trian. These forms will not give perfectly accordant results (especially with short sides), but the check is useful provided the disagreement with 6 Adj. is less than the uncertainty which circumstances indicate to be possible in 12 or 18 Trian. (see para 86). The scrutinising officer should see that this has been done, and that the discrepancies are not excessive.

The final synopsis, and the corrections to forms 20 and 21 Topo. must be checked.

Note.—The scrutiny of each stage of the computations need not be postponed until all computations are completed, but it should follow some considerable time after each stage is complete. The detection of an error should be an extremely rare occurrence for which the computers should be held blameworthy, and the scrutiny should therefore not take place until the computers have definitely finished with that stage of the computation and have at least proceeded from it to the next. If the scrutinising officer has himself been advising and directing the computers, the greater the postponement the better, in order that his mind may bear more freshly on the matter, and that he may be the more likely to detect any error which may possibly have been committed under his own direction.

Any mistakes found in scrutiny, should be recorded in the history sheet.

89. Miscellaneous instructions for computers.—

(*a*) In geodetic triangulation all azimuths should be computed to two decimals of a second, log sides to 7 decimals, and latitudes and longitudes to 3 decimals of a second.

(*b*) In all Survey of India forms the sign + indicates that the quantity is positive. But the sign − does not definitely indicate a negative quantity: the correct sign must be determined and the − must be converted to + where necessary.

(*c*) In all Survey of India forms latitude and longitude are referred to by the symbols λ & L respectively. In Europe and America the more usual practice now is ϕ & λ. This is an unfortunate source of confusion, which it is hard to remedy. λ & L are at present retained in forms, but for publication of results in Geodetic Reports, Professional Papers, etc., ϕ & λ are recommended.

(*d*) It is essential that computers should work neatly and form their figures carefully. When one figure of a seven-figure number has to be corrected the whole group of 3 or 4 figures in which the correction occurs should be struck out, and the correct figures should be entered above. To provide room for this possibility, figures should be written with a fine pen and should not occupy the whole height of the space provided.

(*e*) Consideration should be given to the order in which the lines of a computation form are filled up: it will not always be best to start at the top and to proceed uninterruptedly to the bottom. Where data have to be abstracted from one form to another the whole of the data should be abstracted at once. Similarly if a number of log sines are required it will be convenient to look them out in approximately the order of the magnitude of the angles concerned.

(*f*) Computers should adapt their habits of computation so as to secure economy of effort. For instance, the looking up of a log sine should follow a definite routine. The actual routine followed will depend on the ability of the computer, since a very skilled computer may be able to carry 9 figures in his head with confidence, while others cannot. The following is an example of a possible routine for looking up a log sine in Shortrede's tables. Look at the degrees, minutes and seconds of the angle required: carrying them in the head, find the correct page and column in the tables: place the forefinger of the left hand below the four figures corresponding to the required second, but do not read it: read the integral part of the log and the first three decimals, glancing along the column to see if a diamond intervenes, and record them on the form. Then read the decimals of a second of the angle required, and look at the table of differences: mentally compute the required difference, apply it to the four figures marked by the finger, and record. A practised computer (with another computer working in duplicate with him) will not verify the log: by so doing he will only waste time and develop careless habits.

Similarly, effort may be saved by a convenient (and habitual) placing of the computation form, log-book and inkpot. Each computer must develop his own system, in consultation with a computer of greater experience: but the Officer in charge should enquire into the systems of his computers and rectify them if they are badly thought out.

APPENDICES

APPENDIX I

GRAZING RAYS

If a ray of light passes through air in which the horizontal temperature gradient at right angles to the direction of the ray is $\frac{dT}{ds}$, it can be shown that the resulting curvature of the ray is $1{,}000 \frac{P}{T^2} \cdot \frac{dT}{ds}$ seconds of arc per foot of ray, where P is the atmospheric pressure in inches of mercury, T is the temperature in °F measured from the absolute zero, and $\frac{dT}{ds}$ is measured in °F per foot.

Taking reasonable values of P and T, viz. P = 28 inches and T = 523° (65 °F), we have:—

$$\text{Curvature} \doteqdot \frac{1}{10} \frac{dT}{ds} \text{ seconds per foot.}$$

To take a concrete case, consider a ray which passes 3,000 feet horizontally and 1,000 feet vertically distant from a hill-side parallel to it. At midday the hill-side is probably 5° or 10° F (say $7\frac{1}{2}$°) hotter than the air at the ray, so that the average gradient in the horizontal 3,000 feet between the ray and the hill will be $7\frac{1}{2}$° per 3,000 feet. The gradient will of course be highest close to the hill, but it is possible that it may still be as much as 1° F per 1,000 feet near the ray, so that $\frac{dT}{ds} = 0{\cdot}001$.

The curvature of the ray is then $\frac{1}{10} \times {\cdot}001 \left(= \frac{1}{10{,}000}\right)$ seconds per foot, so that if the ray grazes the hill-side for a distance of 10,000 feet the total curvature of the ray will be 1″. If the graze is near one end of the ray, this curvature will take the form of an almost equal error in the angle measured at that end : if it is in the middle, errors half as great will arise at either end. In any case the triangular error will be affected by the whole amount of 1″.

The figure of 1° F per 1,000 feet given for $\frac{dT}{ds}$ is probably larger than will often be found at such a distance from a hill, but very much larger gradients may be found within a few feet of heated rock. At 7 feet above ordinary ground a vertical gradient of 250° F per 1,000 feet has been recorded*, and a similar gradient could be expected 7 feet to one side of a vertical rock. Under such circumstances it would only be necessary for the graze to be 40 feet long in order to produce a curvature of 1 second.

In practice it will never be possible to estimate $\frac{dT}{ds}$ and to apply any correction, but the foregoing analysis does indicate some useful points, which can be kept in mind.

(1) That a graze over a horizontal hill-top is less serious than a graze along the side of a sloping hill.

(2) That a graze for a short distance, as over a sharp spur, is less serious than a long graze by the side of a hill which lies parallel to the ray for many miles.

(3) That if some observations are made during the day, and some during the night when $\frac{dT}{ds}$ is likely to be of opposite sign, the sign of the error is likely to be reversed and partial cancellation may be expected.

(4) A close graze is, of course, more serious than a distant one.

Both the concrete examples given are extreme cases of grazes, the first as regards length, and the second as regards closeness: and in each case the value taken for $\frac{dT}{ds}$ has been the greatest that may be feared rather than the average to be expected. Provided that care is taken to avoid extreme conditions, it may be concluded that the risk of really serious error is not great.

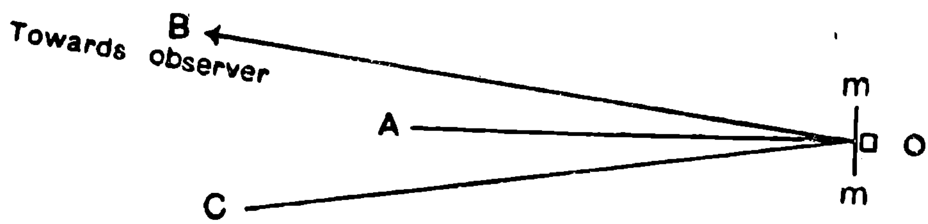
* See Professional Paper No. 22, page 29.

APPENDIX II

USE OF HELIO WITH 100-FOOT MAST

[ABSTRACTED FROM CAPTAIN G. H. OSMASTON'S REPORT IN GEODETIC REPORT, VOL. III.]

To use the mirror as a heliograph, all that is necessary is to direct the sun's rays on to the upper mirror by means of a second large mirror on the ground, from a point where the observer's station is visible in the upper mirror.



In the above figure the mast is shown in plan at O, and m m represents the mirror at the mast-head. The problem is to find a point on the ground, from which the observer's station can be seen in the mirror m m. First, the position A on the ground, where one's own reflection can be seen in the upper mirror is found and marked with a peg. Using a plane-table or compass, the line O B, towards the observer, is then laid out. Now, in order to send light to the observer, using the mirror m m, clearly the source of light must be situated somewhere on the line O C, where the angle COA = the angle AOB. This line O C can easily be marked on the ground, and also a point found on it, from which the horizon is visible. From this position, or very near to it, the observer's station will be visible in m m.

The procedure adopted was for the observer to shine a powerful helio in the direction of the mast, at a pre-arranged time. The lamp-man at the mast, having already found or been shown the approximate position from which he should see the observer's station, soon picked up the observer's light (by the method just described). Having once seen it, there was no further difficulty. He adjusted his ground helio at the same place, and kept the light of the sun focussed on the top mirror, as long as required.

APPENDIX III

AZIMUTH OBSERVATIONS

1. Object.— Astronomical observations for azimuth at primary triangulation stations are required for two purposes, viz. :—

(*a*) To control the accuracy of the triangulation. As stated on page 2 (foot note), this is only possible if the astronomical longitude is also observed. Stations at which both these observations are made are known as Laplace stations.

(*b*) To measure the deviation of the vertical. If the triangulation is adequately controlled by Laplace stations, the discrepancy between the azimuth of a station as observed astronomically and as brought up by the triangulation is a measure of the east-and-west component of the deviation of the local vertical. A knowledge of the deviation is sometimes necessary for the computation of the triangulation itself (see para 71), and is always useful for scientific purposes. The formula is :—

$$\text{Deviation} = (A - G) \cot \lambda \; *,$$

where A = astronomical azimuth,
G = geodetic azimuth,
λ = latitude,
and westerly deflections of the plumb-line are reckoned positive.

2. Frequency and Accuracy.—As stated in para 5, Laplace stations should be provided at intervals of about 200 to 500 miles. The Indian triangulation is at present (1931) seriously deficient in Laplace stations. As regards accuracy, it is desirable that at Laplace stations the precision of the observed azimuth should be about equal to that of the ordinary angles of the triangulation: there is obviously little object in exceeding this accuracy, and little harm will result if the azimuth observation is slightly weaker. A probable error of $0''\cdot4$ is a suitable standard.

For the correction of the angles of the triangulation it is only necessary to observe azimuth (and also latitude) at stations where

---

* G should not be taken direct from the computations or triangulation pamphlets unless adjustment has been made on Laplace stations. See Professional Paper No. 16, page 165. A correction δG is necessary to allow for the distribution of the closing error at Laplace stations.

the rays have exceptionally high elevations or depressions. It will be desirable if elevations exceed + 2° or − 2°. For this purpose it suffices to measure the deviation correct to 5″. On account of the factor cot λ in the formula for the deviation, the azimuth must be measured with an accuracy greater than 5″, but a probable error of 0″·7 will usually suffice. In low latitudes this accuracy will hardly be sufficient (near the equator the deviation is indeterminate by this method), but since the geodetic azimuth will generally be liable to an error of one or two seconds, there is little object in observing the astronomical azimuth more carefully. In low latitudes, if a knowledge of the east-and-west deviation is essential, it must be obtained by longitude observations.

For scientific purposes it is desirable to measure the deviation correct to one or two seconds at as frequent intervals as possible. In particular the north-and-south component (as given by latitude observations) should if possible be measured at every station of one flank* of all meridional series, and the east-and-west component at every station of one flank of all longitudinal series. In oblique series it is desirable to measure both components. Observations have not been made with such frequency in the past, but in late years the ease of observation has been so much increased that there is little reason why it should not be done in future.

It should be noted that frequent observations of rather a low standard of accuracy are more useful than infrequent observations of greater accuracy. A probable error of one second is perfectly satisfactory (although 0″·5 might be preferred). South of latitude 30°, it will be difficult to obtain a *p.e.* of less than 1″ for east-and-west deflections deduced from azimuth observations, and it will be necessary to be content with such a *p.e.* as 1½″ (say 0″·7 in the azimuth observation). South of latitude 20°, azimuth observations are of doubtful value for this purpose.

3. Programme of observation.—In the past the Survey of India have measured azimuth by the observation of circum-polar stars near elongation. This entails a long and trying programme, and the method now adopted is the observation of Polaris at any hour angle. Rough observations for time are of course also necessary.

At a Laplace station the angle between Polaris and the station adopted as referring mark should be measured three times on F.L. and three times on F.R. on each of 10 zeros. At any other station it will usually be sufficient to observe one F.L. and one F.R. on each of 10 zeros.

* i.e. in a straight north and south line. If the series is crooked, the best line may change from one flank to another.

Time should be observed by a pair of east-and-west stars before the azimuth observation and a pair after (one F.L. and one F.R. on each), but if the observations extend to a second day (as is desirable in the case of a Laplace station) one pair will suffice on the second night, provided the rate of the chronometer is already known to within 2 and 3 seconds a day. It should be noted that an error of 1^s of time results in an error of only $\frac{1}{4}''$ of arc when Polaris is at transit, and at other times correspondingly less.

When making azimuth observations all the usual precautions must be observed with regard to the direction of swing and avoidance of over shooting, but it will be permissible to make only one intersection of Polaris at each measure, instead of the usual two or three*.

The routine of the observation is then as follows:—

(1) After preliminary rotation of the telescope intersect the R.M. and record its readings in the usual way. (According to the direction of swing the R.M. may of course come before or after Polaris).

(2) Intersect Polaris, being very careful not to over shoot it beyond the range of the slow motion screw.

(3) When satisfied with the intersection, start the stop-watch. (Polaris moves too slowly for it to be preferable to let it make the intersection by its own movement).

(4) Read the east-and-west bubble.

(5) Stop the stop-watch on some convenient reading of the chronometer (e.g. $12^h\ 28^m\ 35^s$), and read the time of intersection thus:— $12^h\ 28^m\ 35^s$ *minus* $28^s{\cdot}7$ ($28^s{\cdot}7$ being the stop-watch reading).

(6) Read the horizontal circle.

It is important to note that the readings of the east-and-west bubble (on the lower plate) while intersecting Polaris are essential. The instrument should be carefully levelled and the mean bubble correction kept small to avoid error due to doubt in the value of one division.

4. Computation.—The results are computed on form 25 Topo. (see fig. 40). In the case of observations at Laplace stations each observation must be computed separately, but at other stations the mean F.L. and F.R. angles may be computed with the mean times.

* In the case of a Wild theodolite the seconds of the "reduced reading" will thus be obtained by simply doubling the actual reading.

At Laplace stations also, a small correction must be applied, which is due to an approximation in the formula on which the form is based (see Table 43 Geod. Aux. Tables, Part IV).

Each deduction* must be corrected for the transverse bubble, the correction to the deduced azimuth being $+\left(\frac{\Sigma\,(E-W)}{n}\right) d \tan \lambda$, where n is the number of bubble readings (E & W counting as one each), d is the value of one division of the level, and λ is the latitude. The above formula supposes the bubble to be graduated outward from the centre.

Time is computed on form 15 Topo. One deduction should be made for the mean of the F.L. and F.R. observations of each star.

* It is of course possible to compute a mean correction to the whole programme at once. But unless the levelling has been kept very steady this will result in a disappointingly large range of variation on the different zeros.

15 PARTY (Triangulation) SEASON 1930-31

Computation of Azimuth from Polaris, *time and latitude being known.*

1	Deduction Number		Zero 18°			Zero 36°			
2	Name of station	Keng	tung Base	South	Keng	tung Base	South.		
3	Long. in time by 34 Sur.(1) = l	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s
4	Polaris observed $\frac{\text{East}}{\text{West}}$ of meridian(7)								
5	Hour and date of observation	19	19	19	19	19	19	19	19
6	Local Mean Time of obsn. = m(2)	h m s	h m s	h m s	h m s	h m s	h m s	h m s	h m s
7	Sidereal Time(3) or R(5)								
8	Sum								
9	A(4) for ($m - l$) from 22 Sur.(1)	–	–	–	–	–	–	–	–
10	Sum = L. S. T. of observation(5)	6 00 04·8	6 02 49·5	6 06 09·5	6 56 45·7	7 00 16·3	7 03 48·8		
11	R.A. of Polaris from N.A.	1 36 18·4	1 36 18·4	1 36 18·4	1 36 18·4	1 36 18·4	1 36 18·4	1	1
12	line 10 − line 11 = t = hour angle	4 23 46·4	4 26 31·1	4 29 51·1	5 20 27·3	5 23 57·9	5 27 30·4		
13	t in arc by 45 Sur.(1)	65° 56′ 36″	66° 37′ 47″	67° 27′ 47″	80° 06′ 50″	80° 59′ 29″	81° 52′ 36″	° ′ ″	° ′ ″
14	N. P. D. = 90° − Declination = Δ	1 03 38·8	1 03 38·8	1 03 38·8	1 03 38·8	1 03 38·8	1 03 38·8	1	1
15	Δ in seconds	3818·8″	3818·8″	3818·8″	3818·8″	3818·8″	3818·8″	″	″
16	log Δ in seconds	3 581 93	3 581 93	3 581 93	3 581 93	3 581 93	3 581 93	3	3
17	log cos t	$\bar{1}$ 610 28	$\bar{1}$ 598 43	$\bar{1}$ 583 82	$\bar{1}$ 234 75	$\bar{1}$ 194 75	$\bar{1}$ 150 16		
18	Sum = log a in seconds	3 192 21	3 180 36	3 165 75	2 816 68	2 776 68	2 732 09		
19	a in seconds	1557″	1515″	1465″	656″	598″	540″	″	″
20	a(6)	+ 0° 25′ 57″	+ 0° 25′ 15″	+ 0° 24′ 25″	+ 0° 10′ 56″	+ 0° 09′ 58″	+ 0° 09′ 00″	– ° ′ ″	– ° ′ ″
21	Latitude of station = λ	21 18 23	21 18 23	21 18 23	21 18 23	21 18 23	21 18 23		
22	Sum = λ + a	21 44 20	21 43 38	21 42 48	21 29 19	21 28 21	21 27 23		
23	log sec(λ + a)	0 032 04	0 032 00	0 031 96	0 031 29	0 031 24	0 031 19		
24	log sin t	$\bar{1}$ 960 54	$\bar{1}$ 962 82	$\bar{1}$ 965 50	$\bar{1}$ 993 50	$\bar{1}$ 994 61	$\bar{1}$ 995 62		
25	log Δ in seconds (from line 16)	3 581 93	3 581 93	3 581 93	3 581 93	3 581 93	3 581 93	3	3
26	Sum = log A in seconds	3 574 51	3 576 75	3 579 39	3 606 72	3 607 78	3 608 74		
27	A in seconds	3754·2″	3773·5″	3796·5″	4043·2″	4053·0″	4062·0″	″	″
28	A(7)	− 1° 02′ 34·2″	− 1° 02′ 53·5″	− 1° 03′ 16·5″	− 1° 07′ 23·2″	− 1° 07′ 33·0″	− 1° 07′ 42·0″	– ° ′ ″	– ° ′ ″
29	180° + A = Az. of Polaris from S.	178 57 25·8	178 57 06·5	178 56 43·5	178 52 36·8	178 52 27·0	178 52 18·0		
30	Angle between R.M. and Polaris	− 48 30 07·5	− 48 29 44·3	− 48 29 22·4	− 48 25 25·0	− 48 25 12·3	− 48 25 04·5		
31	Azimuth of R.M. from S(8)	130 27 18·3	130 27 22·2	130 27 21·1	130 27 11·8	130 27 14·7	130 27 13·5		
32	Correction for approximation in formula	+ 0 00 00·1	+ 0 00 00·1	+ 0 00 00·1	+ 0 00 00·1	+ 0 00 00·1	+ 0 00 00·1		
33	Level correction	+ 0 00 01·8	+ 0 00 00·9	+ 0 00 00·9	+ 0 00 03·5	− 0 00 00·3	+ 0 00 01·8		
34	Corrn. for graduation error of theodolite	+ 0 00 00·5	+ 0 00 00·5	+ 0 00 00·5	+ 0 00 01·3	+ 0 00 01·3	+ 0 00 01·3		
35	Sum = Corrected azimuth of R.M. from S	130 27 20·7	130 27 23·7	130 27 22·6	130 27 16·7	130 27 15·8	130 27 16·7		

(1) Auxiliary Tables, 5th Edn., Part III. (2) e.g. at 6 a. m. enter 6 hrs., at 6 p. m. enter 18 hrs. Correct chronometer time by chronometer error if appreciable. (3) For G.M.T. 0 hour on date of observation. Sidereal time from N.A. (Complete) R from N.A. (Abridged). (4) A is + or − according as m is *greater* or *less* than l. (5) Increase L.S.T. by 24 hrs. if this is *less* than R.A. of Polaris. (6) a is − when t (line 13) lies *between* 90° and 270°; otherwise +. (7) A is + or − according as Polaris is E. or W. of meridian. Polaris is $\frac{W}{E}$ of meridian (line 4) according as t (line 13) is $\frac{0° - 180°}{180° - 360°}$ (8) Verify from diagram in Angle Book.

Computed and compared by C.B.M. and S.K.S. 3-9-1931.

APPENDIX IV

LATITUDE OBSERVATIONS

As stated in Appendix III it is desirable to observe the astronomical latitude at stations where the elevation or depression exceeds 2°, and also at all stations along one flank of a meridional series. The accuracy to be aimed at is a probable error of about one second. This can be obtained without difficulty by the observation of two north and two south circumpolar stars, Polaris out of meridian being used for one of the north stars. Four F.L. and four F.R. observations of each star will suffice, of which not more than three should be on the same side of the meridian. The level on the telescope must, of course, be recorded at each intersection.

Time should be observed by east-and-west stars as in the case of azimuth. Little accuracy is required, but it is desirable to observe four stars, in case of misidentification.

Latitude is computed on form 13 Topo. as explained in Aux. Tables, Part III, Explanation to Table 24 Sur. One deduction should be made for each pair of F.L. & F.R., so that 3 or 4 separate results will be given by each star, from which its accuracy can be judged. Latitude from Polaris is computed on form 14 Topo. separate deductions being made for each pair of F.L. & F.R.

GBPO.—400—1931.